



INCONTRO SUL TEMA:

**Rinforzi FRCC per strutture
in muratura ed in C.A.**



**Criteri di dimensionamento degli
interventi di rinforzo con FRCC
ed esempi di calcolo**

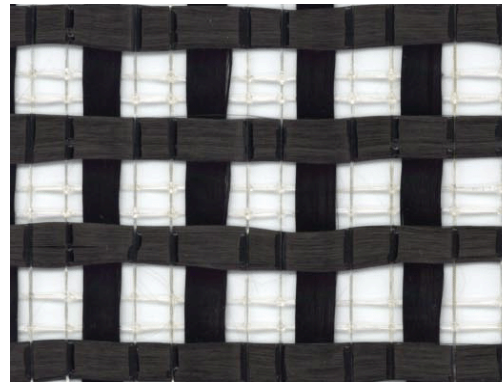


Francesco Focacci

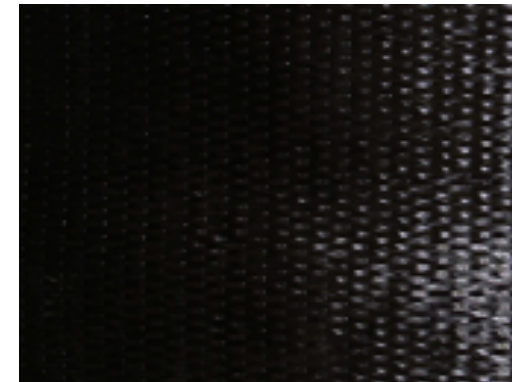
Firenze, giugno 2010

- **CNR DT200:** Istruzioni per la Progettazione, l'Esecuzione ed il Controllo di Interventi di Consolidamento Statico mediante l'utilizzo di **Compositi Fibrorinforzati**

MATRICI CEMENTIZIE



MATRICI EPOSSIDICHE



ALTRI TIPI DI RESINE

“Vanno infine ricordate le **matrici di natura inorganica** (**cementizie**, metalliche, ceramiche, ecc.), il cui utilizzo per la realizzazione di compositi fibrorinforzati in campo civile - **soprattutto quelle cementizie** - sta progressivamente diffondendosi. Pur **non essendo esaminate nel presente documento, il loro impiego è tuttavia ritenuto possibile** a condizione che risulti **suffragato da una documentazione tecnica e da una campagna sperimentale adeguate**, comprovanti un'efficacia almeno pari a quella delle matrici organiche qui trattate”.

- **CNR DT200:** Istruzioni per la Progettazione, l'Esecuzione ed il Controllo di Interventi di Consolidamento Statico mediante l'utilizzo di **Compositi Fibrorinforzati**

➔ Le verifiche degli elementi rinforzati devono essere condotti sia nei riguardi degli SLE che degli SLU.

➔ Dilatazione **ultima di calcolo** del rinforzo

$$\varepsilon_{fd} = \min \left\{ \eta_a \frac{\varepsilon_{fu}}{\gamma_f}, \varepsilon_{f,max} \right\}$$

➔ Delminazione (crisi dell'aderenza)

➔ Rottura del materiale

CFRP



C-FRCM , PBO-FRCM



$$\gamma_f = 1.2 \div 1.35$$

coefficiente parziale di sicurezza (dipendente dalle condizioni di controllo di esecuzione);

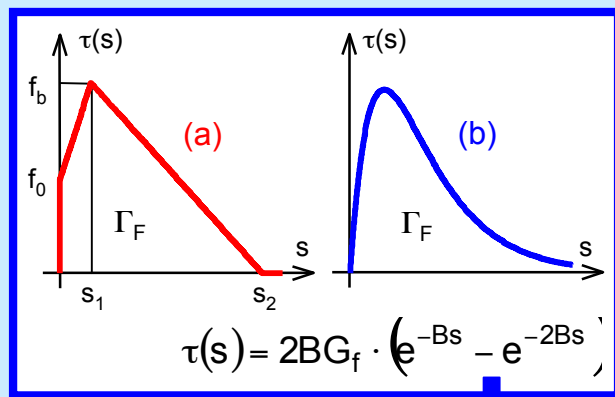
$$\eta_a = 0.5 \div 0.95$$

dipendente dal sistema (tipo di fibre e matrice) e dalle condizioni di esposizione.

DELAMINAZIONE DEL RINFORZO (approccio CNR DT200/2004)

● DESIGN PARAMETERS

➡ Legge aderenza-scorrimento $\tau = \tau(s)$



Dai, J., Ueda, T., Sato, Y.: Development of the Nonlinear Bond Stress-Slip Model of Fiber Reinforced Plastics Sheet-Concrete Interfaces with a Simple Method; Journal of Composites for Construction, ASCE, Vol.9 n. 1, 2005.

$$N_{u,db} = b_f \cdot \sqrt{2 \cdot G_F E_f \cdot t_t}$$

$$\varepsilon_{u,db} = \frac{N_{u,db}}{E_f b_f t_t} = \sqrt{\frac{2 \cdot G_F}{E_f t_t}}$$

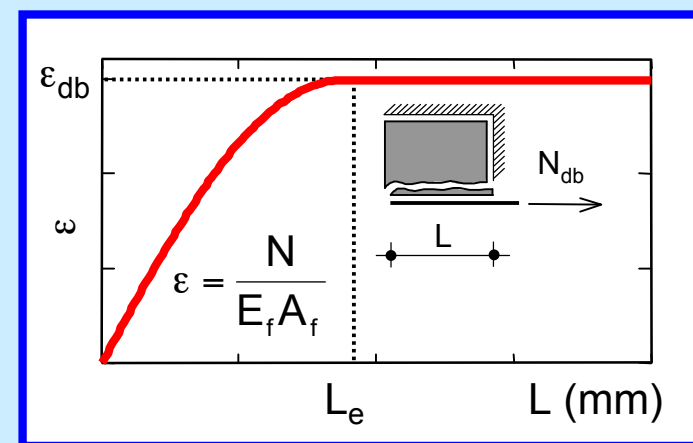
$$\varepsilon_{u,db}$$

Delaminazione **di estremità**
Delaminazione del **rinforzo a taglio**

$$\tau = \tau(s)$$

$$L_e$$

Lunghezza di trasferimento
efficace



➡ Parametro

$$k_{cr}$$

$$\varepsilon_{u,db1} = k_{cr} \cdot \varepsilon_{u,db}$$

Delaminazione **intermedia**

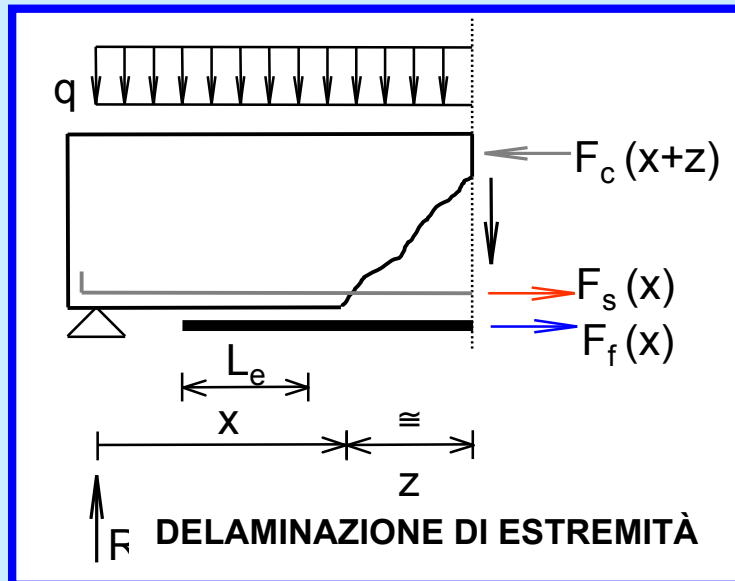
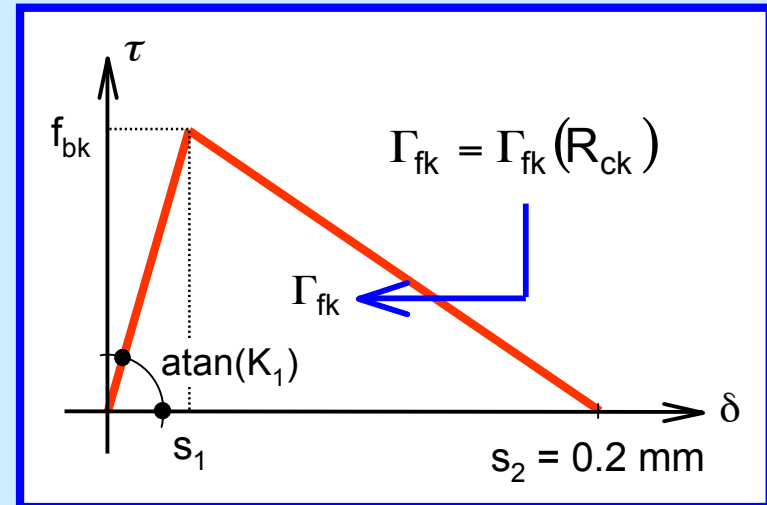
DELAMINAZIONE DEL RINFORZO (approccio CNR DT200/2004)

$$\tau = \tau(s)$$

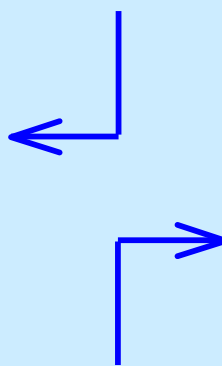
$$F_{\max,d} = \frac{b_f \cdot k_c}{\gamma_{Rd}} \cdot \sqrt{2 \cdot E_f \cdot t_f \cdot \Gamma_{fd}}$$

$$\varepsilon_{f,\max} = \frac{F_{\max,d}}{E_f A_f} = \frac{k_c}{\gamma_{Rd}} \cdot \sqrt{\frac{2 \cdot \Gamma_{fd}}{E_f \cdot t_f}}$$

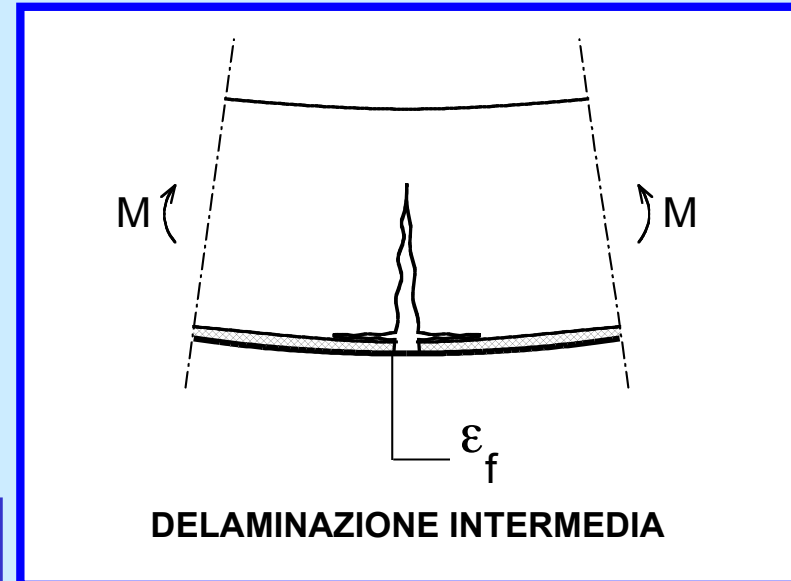
$$L_e$$



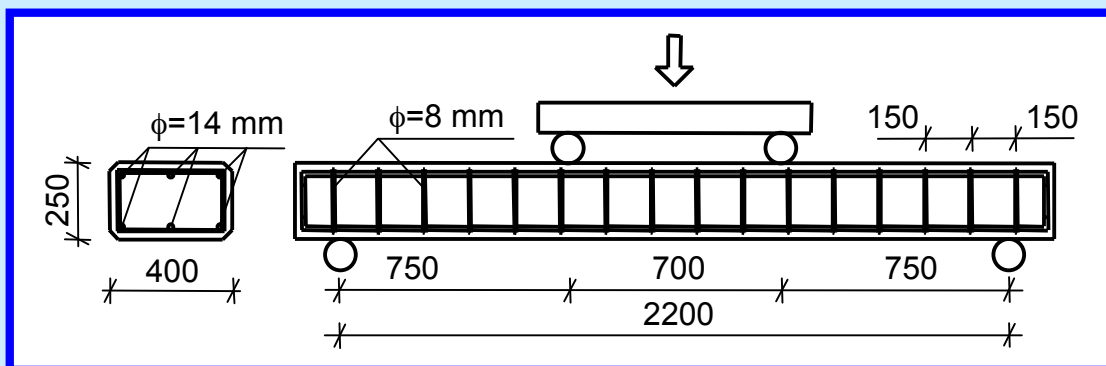
$$\varepsilon_f \leq \varepsilon_{f,\max}$$



$$\varepsilon_f \leq k_{cr} \varepsilon_{f,\max}$$

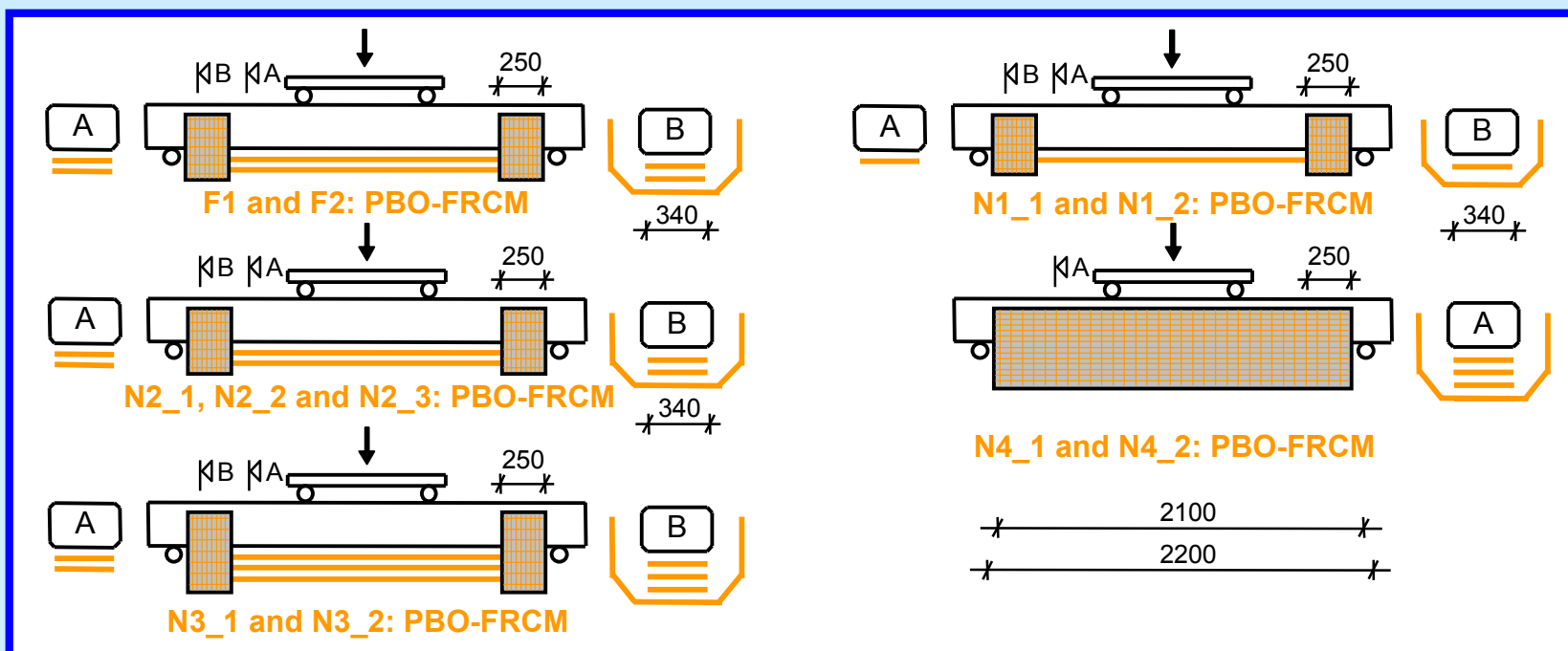


DELAMINAZIONE INTERMEDIA



PROVINI

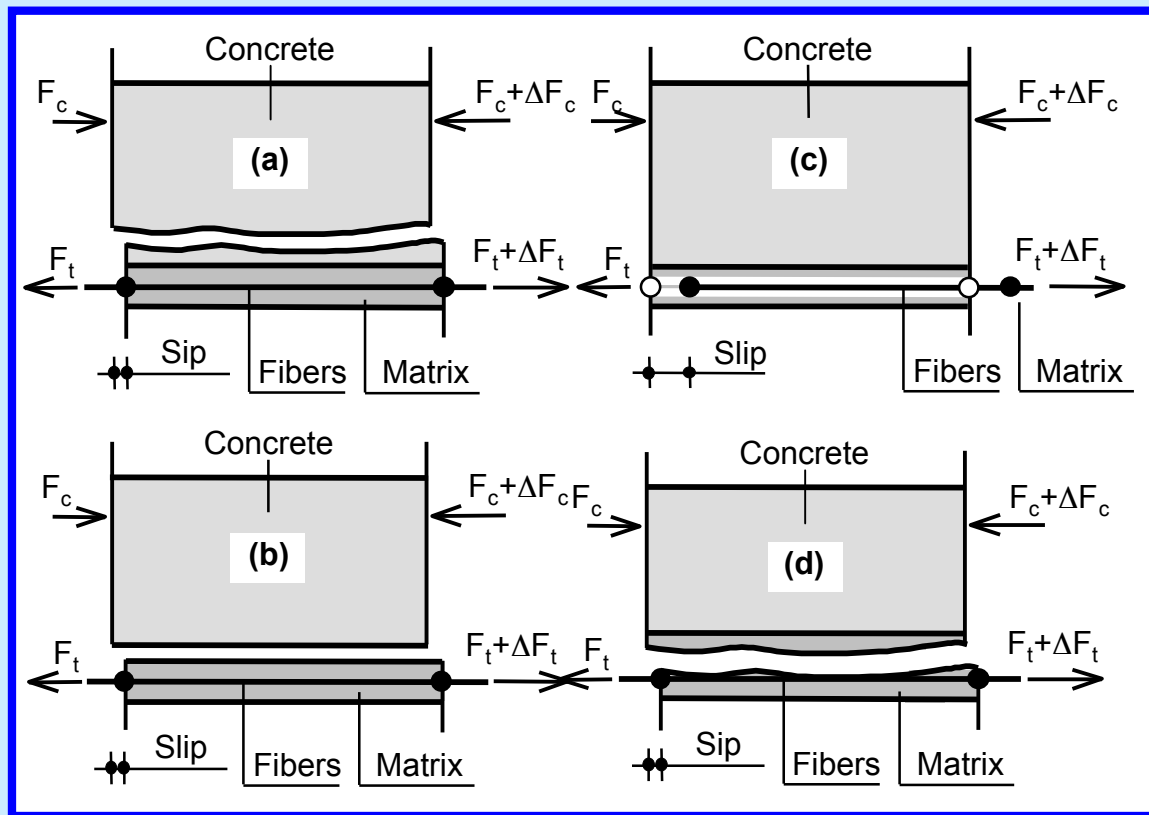
RINFORZO



Concrete compr. strength (cubic) (N/mm ²)	63.66
Steel yielding stress (N/mm ²)	476.17
Steel tensile strength (N/mm ²)	616.73

MATERIALI

MODI DI DELAMINAZIONE



(a) CFRP longer beams



(b) PBO-FRCM shorter beams
C-FRCM longer beams



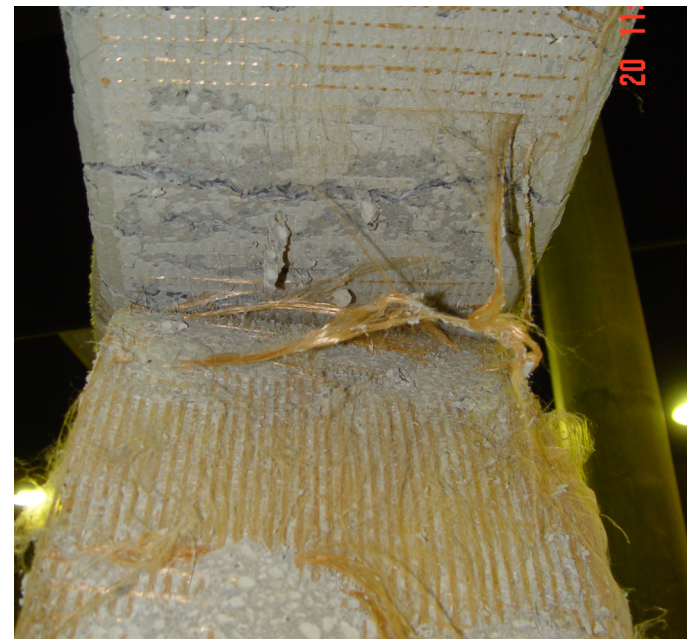
(c) C-FRCM longer beams
PBO-FRCM shorter beams



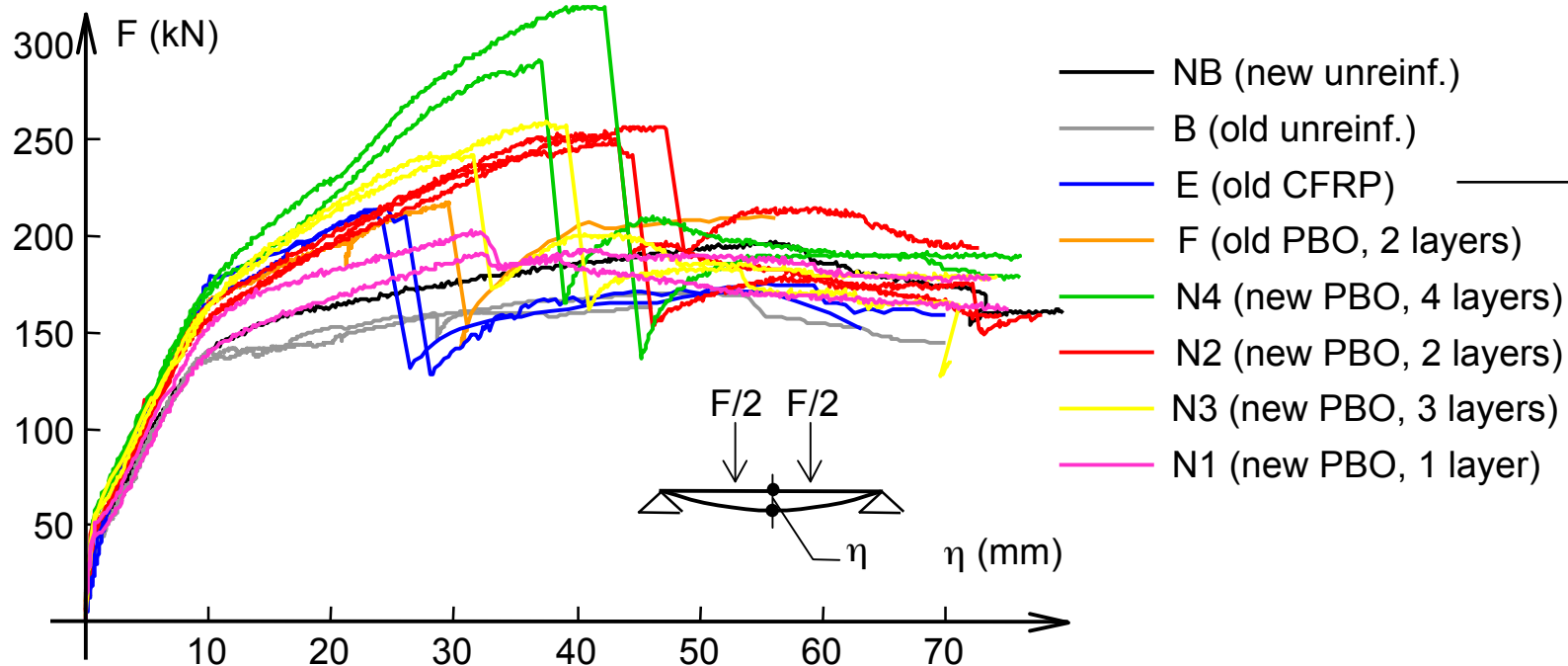
(d) PBO-FRCM longer beams



DELAMINAZIONE



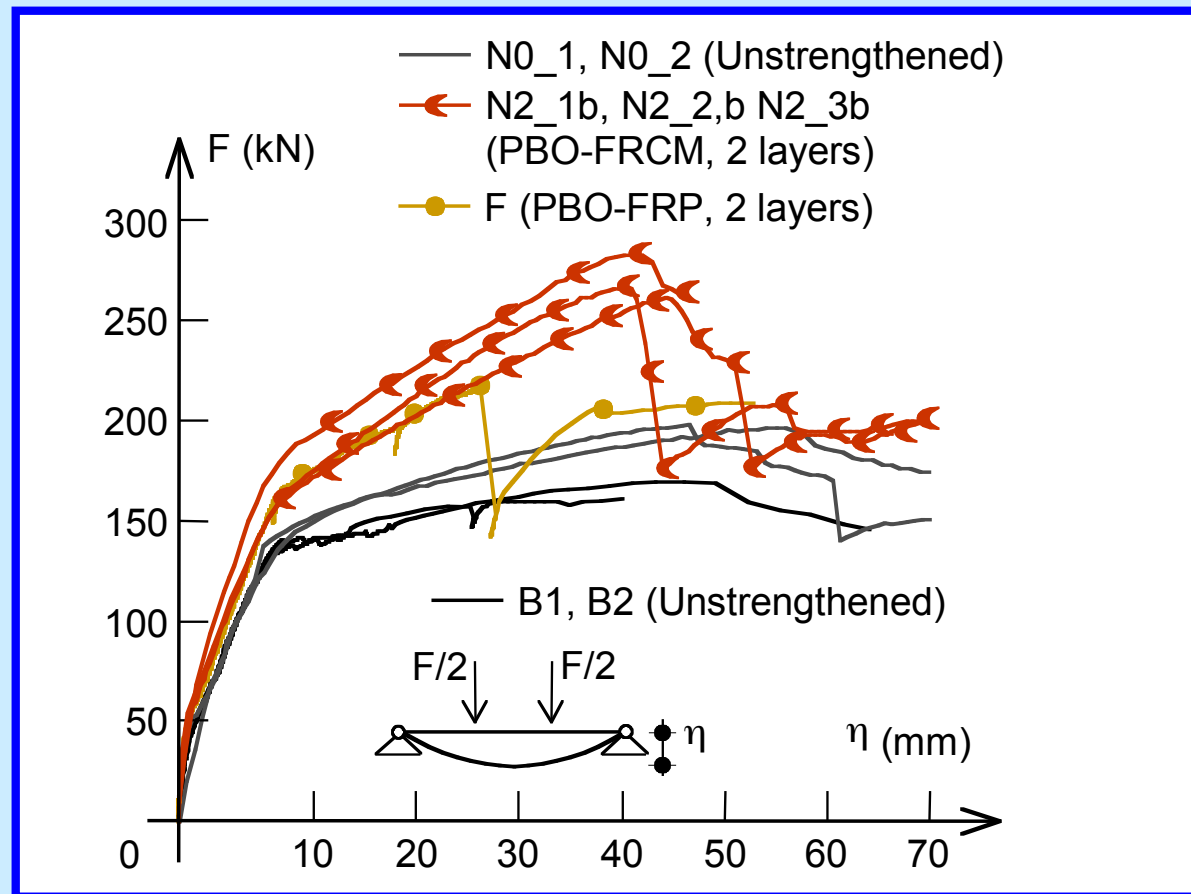
**DIAFRAMMI
CARICO
SPOSTAMENTO**



Specimen	DESCRIPTION	FAILURE MODE	Maximum load (kN)	AVERAGE (kN)	Increasing (%)
B1	Unreinforced	Concrete crushing	163.07	166.54	--
B2			170.00		
E1	1 CFRP layer and U shaped end reinforcing	(a)	215.75	214.91	+ 29.04
E2			214.07		
F2	2 PBO-FRCM layers and U shaped end reinforcing	(b)	217.26	217.26	+ 30.45
7	Unreinforced	Concrete crushing	196.95	196.95	--
N1_1	1 PBO-FRCM layer and U shaped end reinforcing	(b)	192.60	197.73	+ 0.40
N1_2			202.86		
N2_1	2 PBO-FRCM layers and U shaped end reinforcing	(d)	256.92	253.24	+ 28.58
N2_2			253.38		
N2_3			249.43		
N3_1	3 PBO-FRCM layers and U shaped end reinforcing	(d)	243.12	251.41	+ 27.65
N3_2			259.69		
N4_1	4 PBO-FRCM layers, one shaped	(d)	319.68	305.48	+ 55.11
N4_1			291.27		

Stessa sezione di fibre

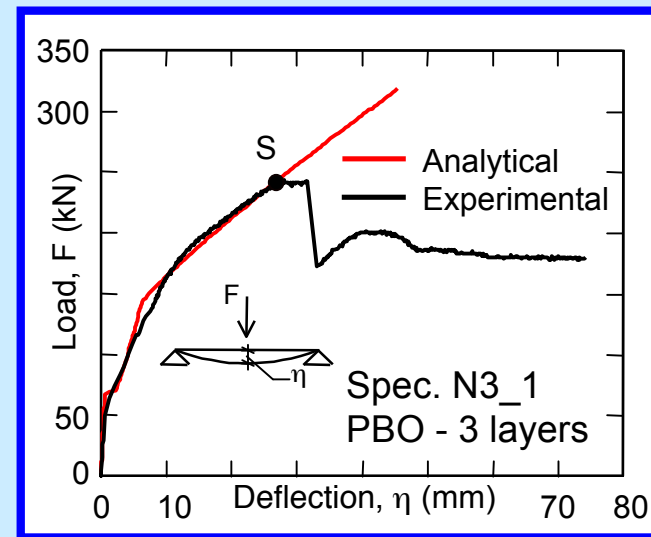
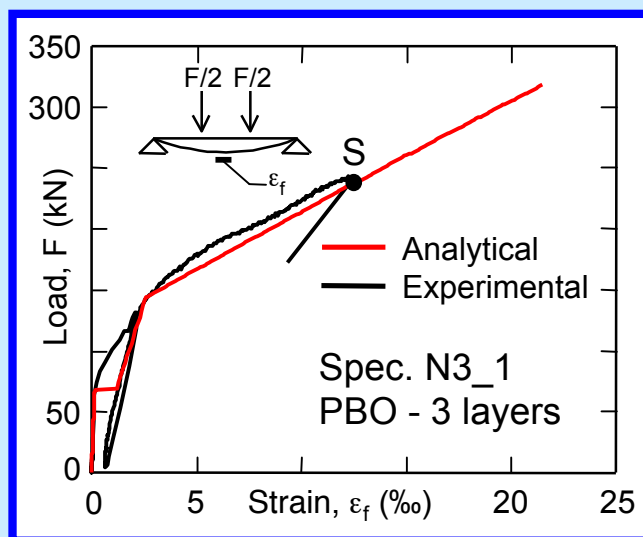
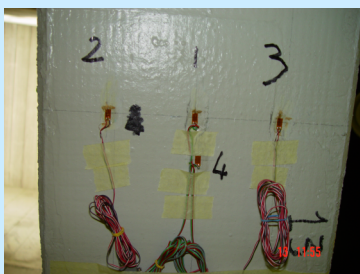
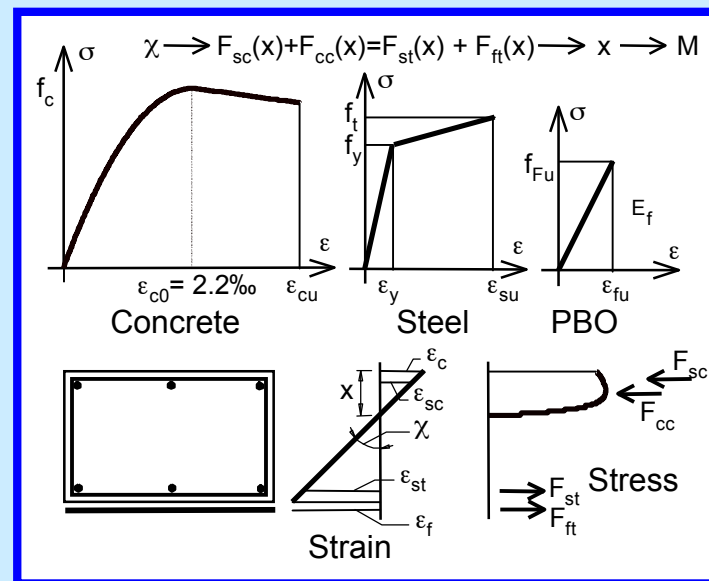
IMPORTANZA DELL'ACCOPIAMENTO FIBRE/MATRICE

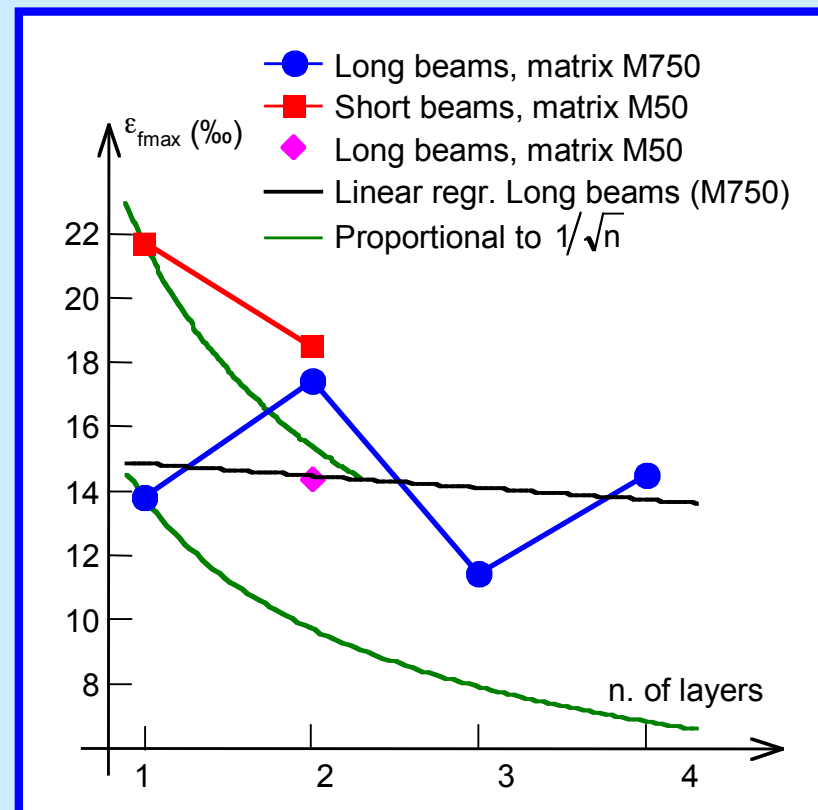
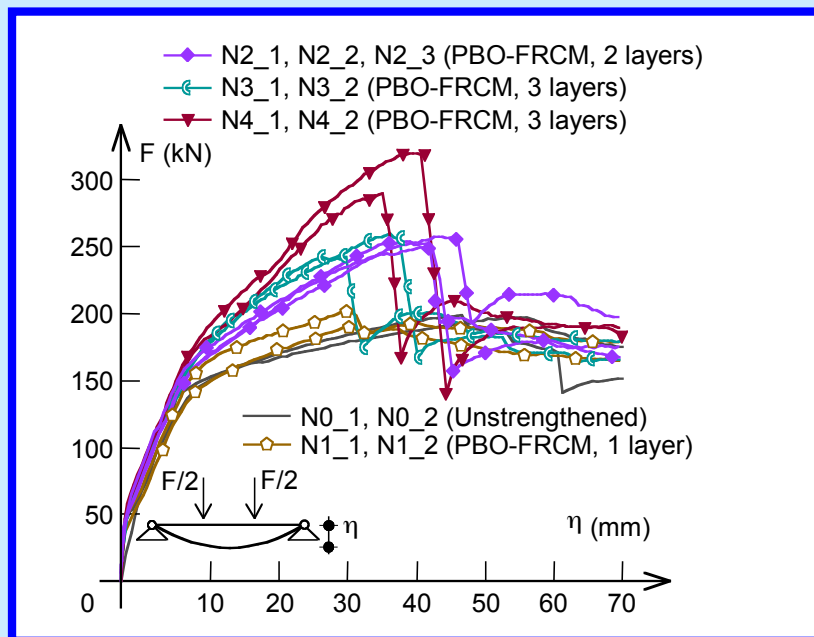


DILATAZIONE MASSIMA

Specimen	Reinforcing material	Strain. ε_{fmax} (‰)
D1 - D2	C-FRCM (2 layers)	8.58 (*)
E1 - E2	CFRP (1 layer)	11.33 (*)
F2	PBO-FRCM (2 layers)	13.77 (*)
E1 - E2	CFRP (1 layer)	11.33 (*)
N1	PBO-FRCM (1 layer)	12.91
N2	PBO-FRCM (2 layers)	18.95 (*)
N3	PBO-FRCM (3 layers)	13.36 (13.26 *)
N4	PBO-FRCM (4 layer2)	15.14 (14.97 *)

(*) Calculated





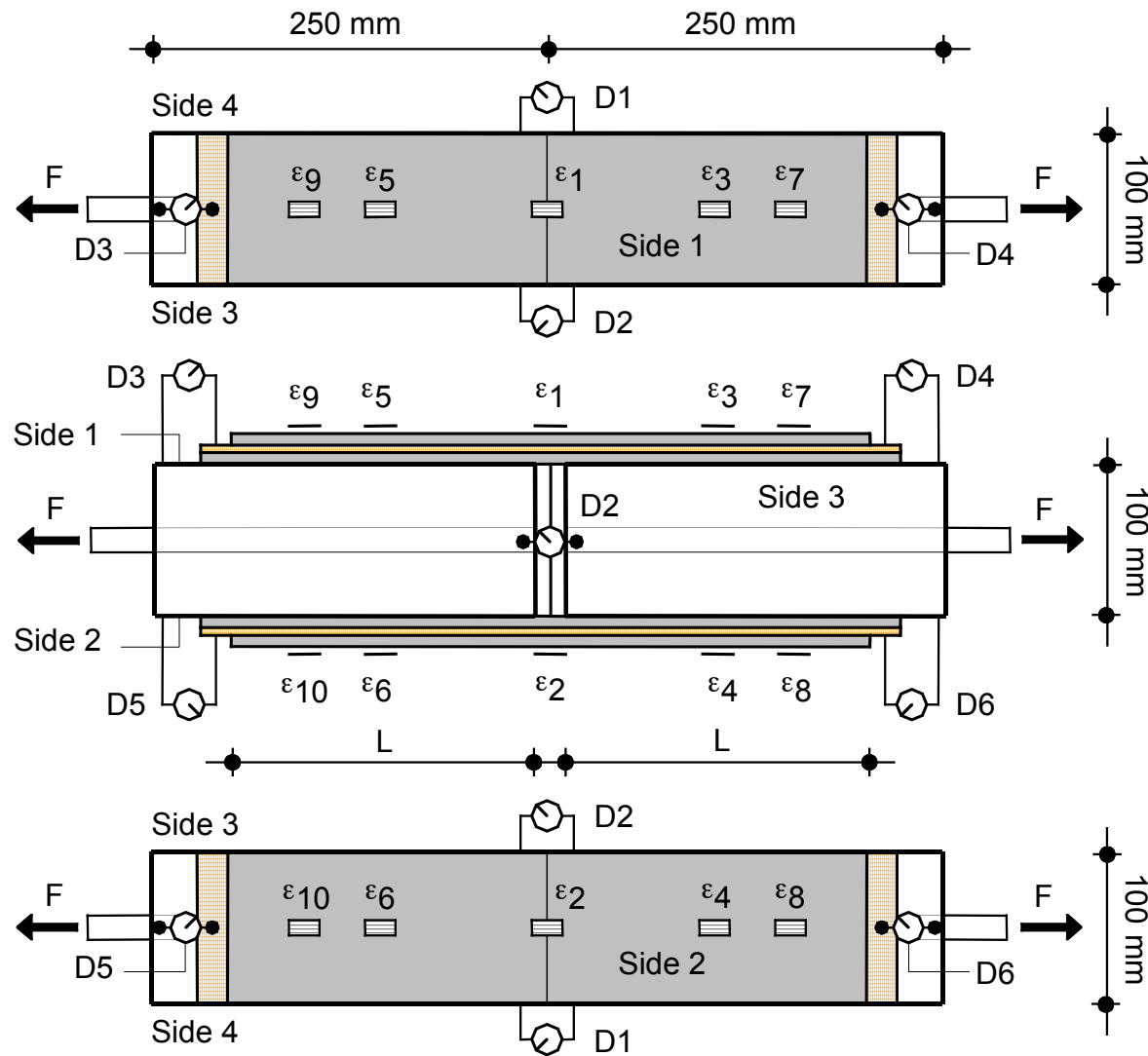
$$N_{u,db} = b_f \cdot \sqrt{2 \cdot G_F E_f \cdot t_t}$$

$$\epsilon_{u,db} = \frac{N_{u,db}}{E_f b_f t_f} = \sqrt{\frac{2 \cdot G_F}{E_f t_t}}$$

$$\epsilon_{u,db}(n) = \sqrt{\frac{1}{n}} \cdot \epsilon_{u,db}(1)$$

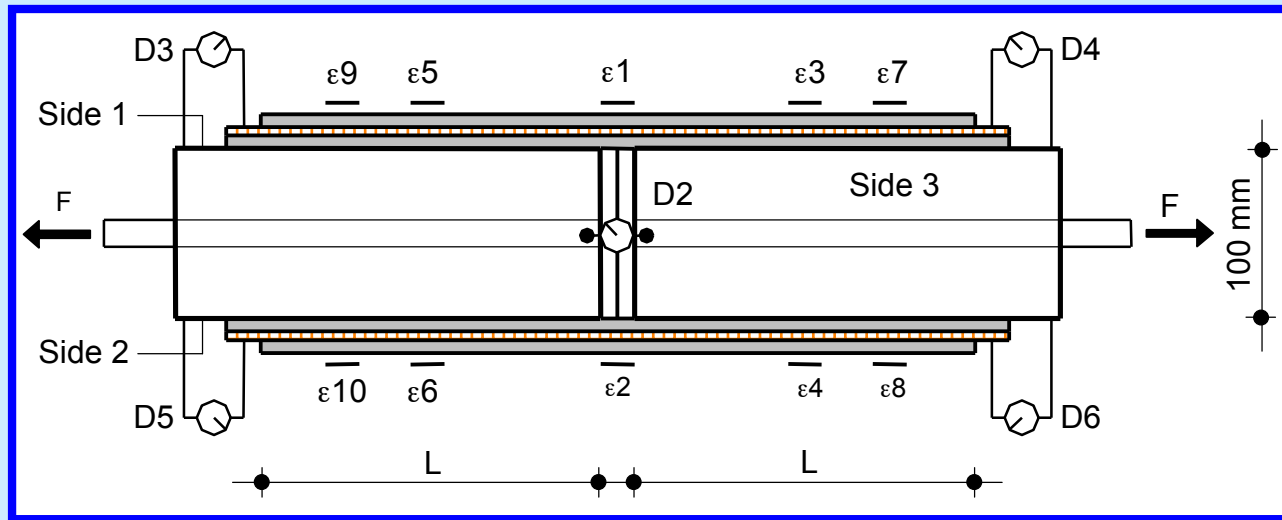
DELAMINAZIONE DI ESTREMITÀ

TEST SETUP



I
U
A
V



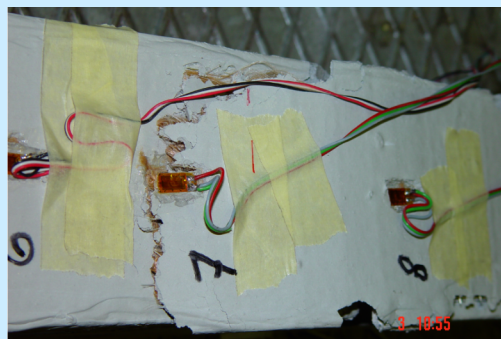
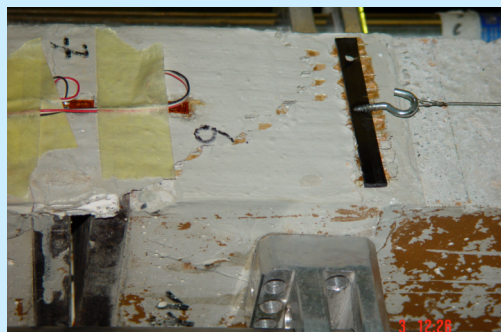


PROVINI

SPECIMEN	L (mm)	LAYERS
S1L50_1	50	1
S1L50_2		
S1L100	100	
S1L150_1	150	
S1L150_2		
S1L200_1	200	
S1L200_1		
S1L250_1	250	
S1L250_2		
S2L100_1	100	2
S2L100_2		
S2L200_1	200	

L. di ancoraggio: 50 ÷ 250 mm
Strati: 1 ÷ 2

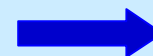
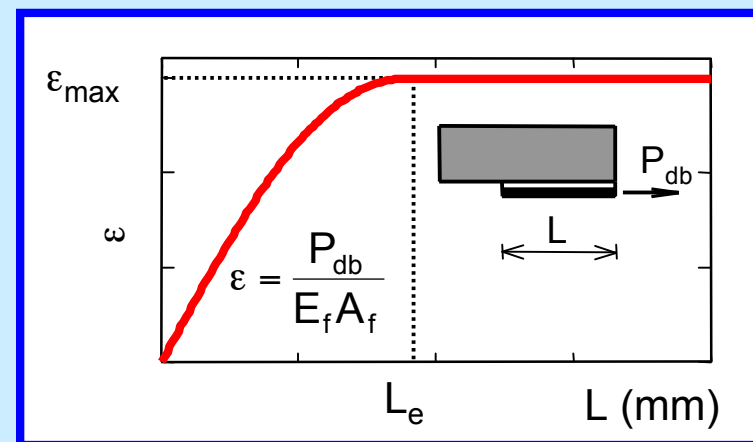
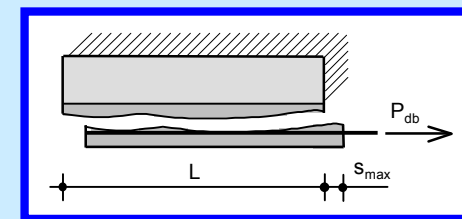
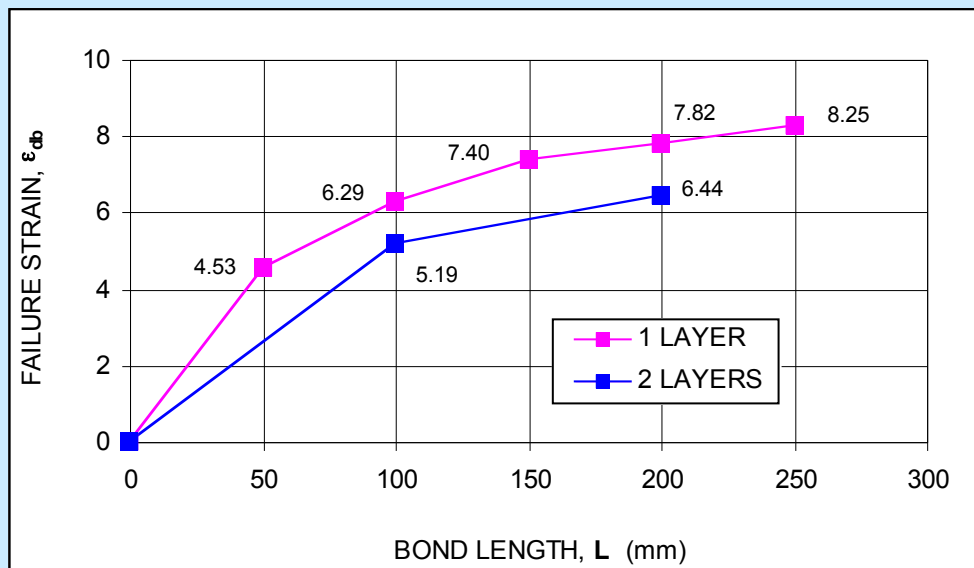
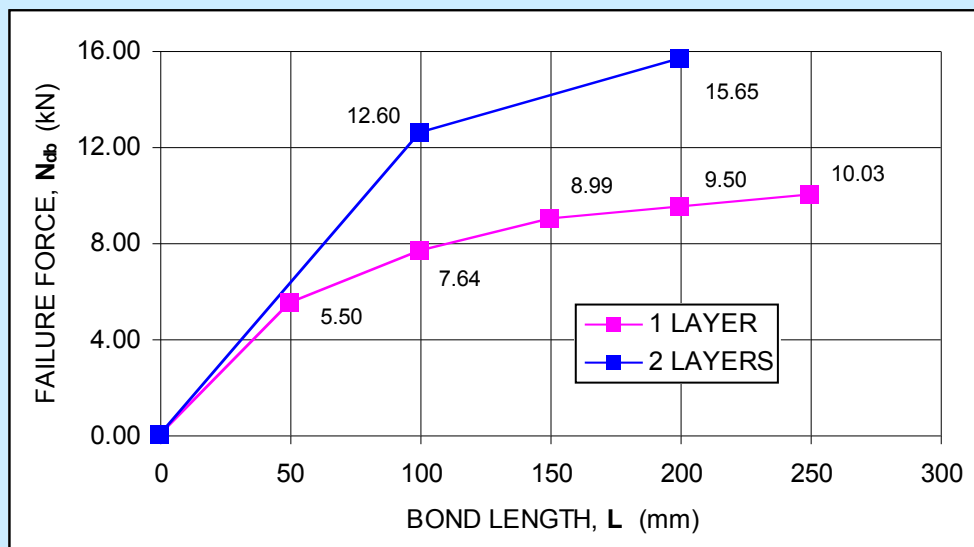




MODO DI DELAMINAZIONE

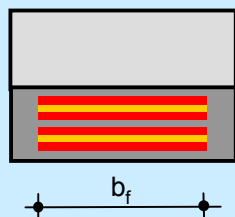
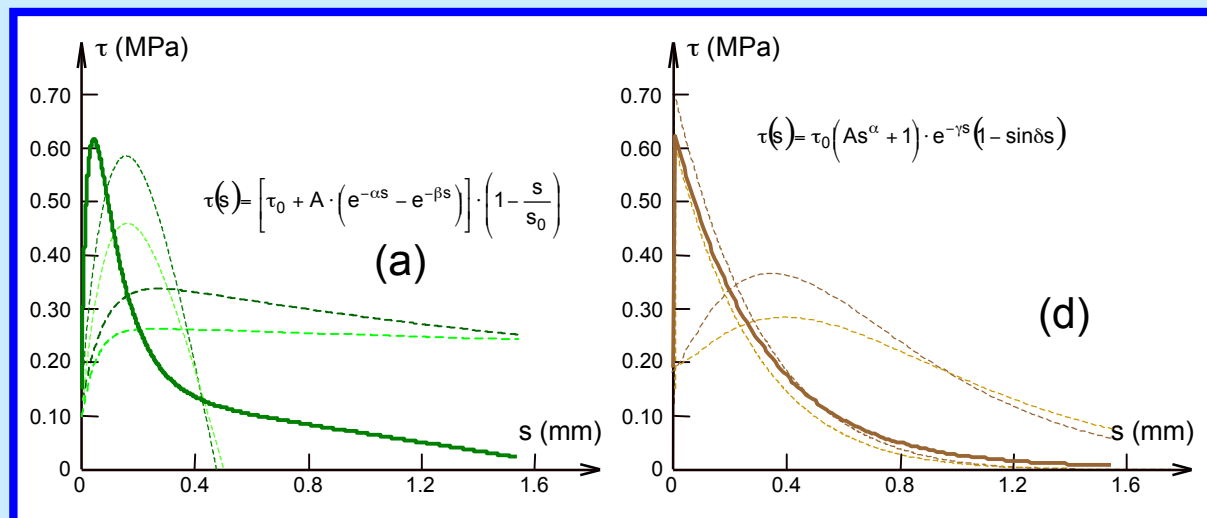


RISULTATI GLOBALI



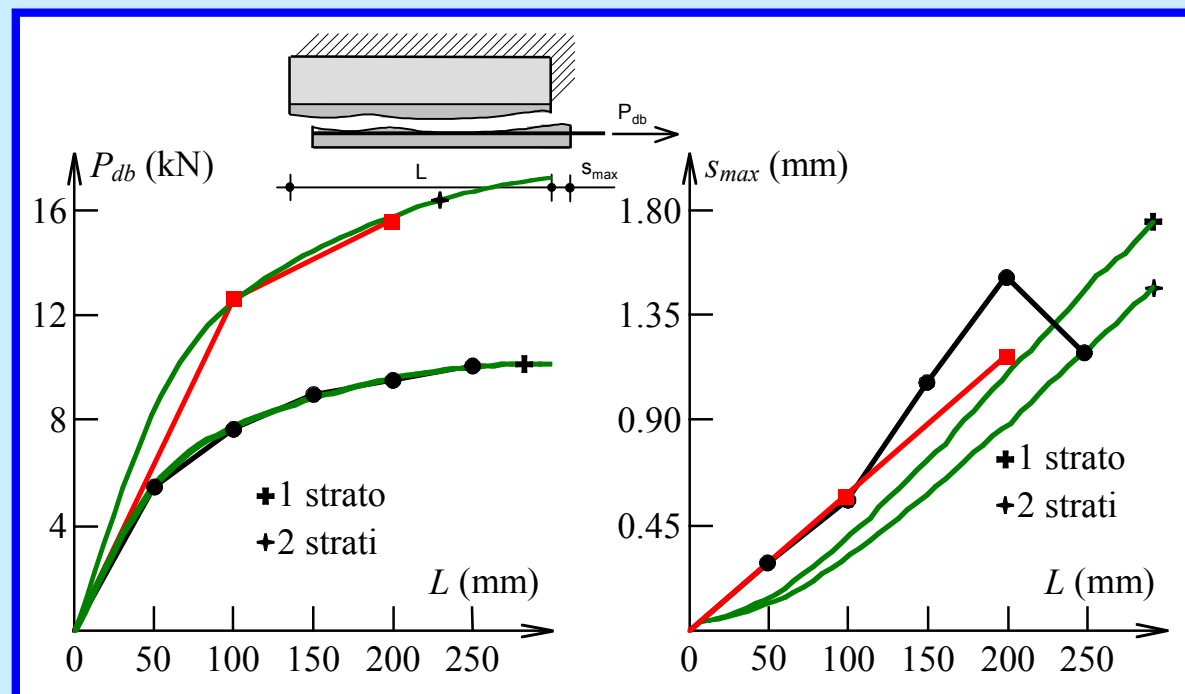
$L_e \approx 250 - 300 \text{ mm}$

GLOBAL RESULTS - LOCAL BOND-SLIP



$$p_f = 2nb_f \cdot k(\eta)$$

$$k(2) = 0.73$$



PROGETTO

● FORMULE GENERALI

$$P_{dbu} = \sqrt{2p_f E_f A_f \cdot G_f}$$

$$\varepsilon_{dbu} = \frac{P_{dbu}}{E_f A_f} = \sqrt{\frac{2p_f G_f}{E_f A_f}}$$

$$\varepsilon_{dbm} = k_{cr} \sqrt{\frac{2p_f G_f}{E_f A_f}}$$

Forza di delaminazione di estremità

Dilatazione di delaminazione di estremità

Dilatazione di delaminazione intermedia

● PARAMETRI DETERMINATI SPERIMENTALMENTE

$$G_f = 211 \frac{J}{m^2}$$

$$p_f(n) = 2nb_f \cdot k(n)$$

$$k(n) = \begin{cases} 1 & \text{se } n = 1 \\ 0.8 & \text{se } 2 \leq n \leq 4 \end{cases}$$

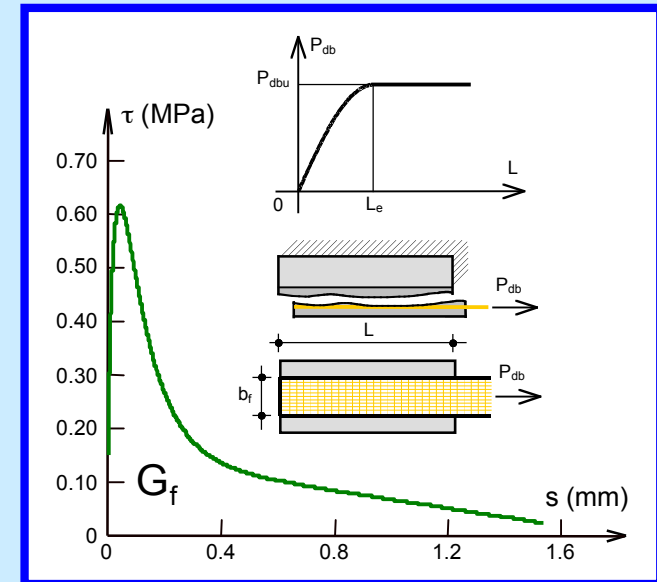
$$k_{cr} = 1.8$$

Energia di frattura

Larghezza di interfaccia

Coefficiente di efficienza

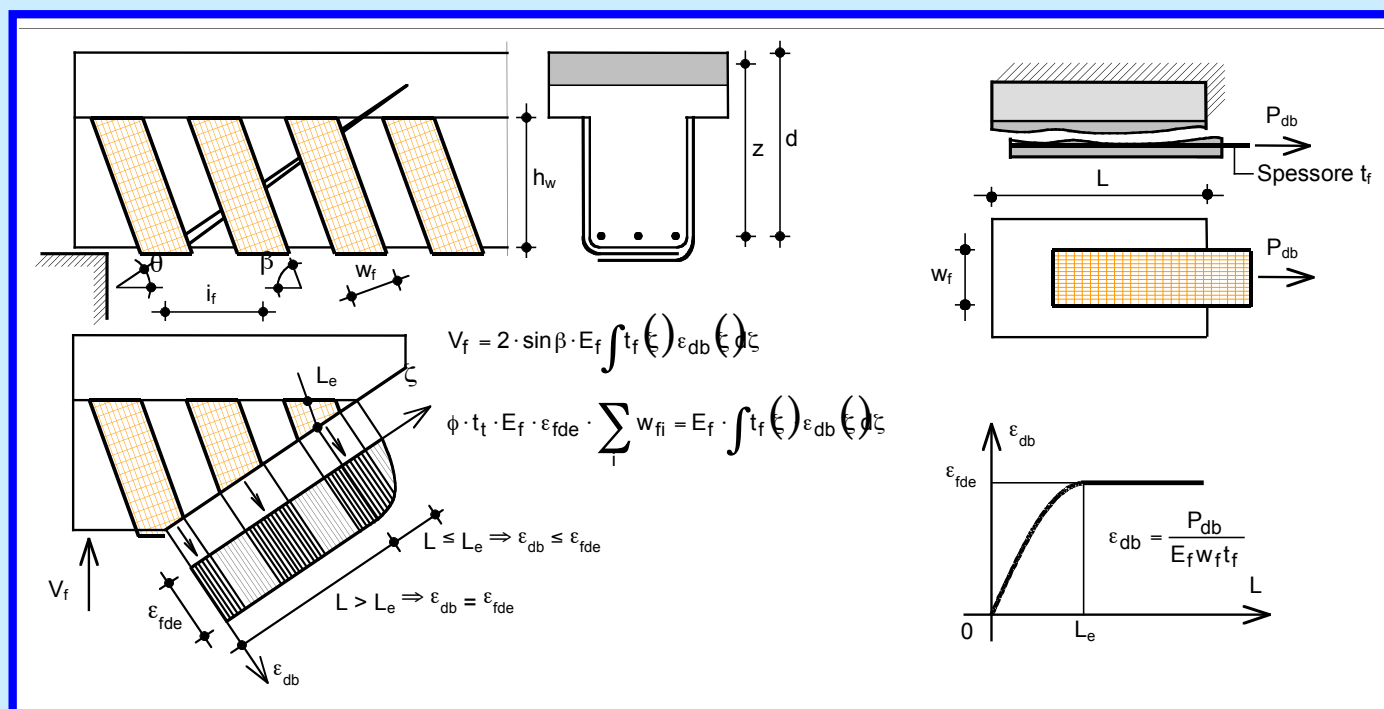
Coefficiente per la delaminazione intermedia



● VALORI DI CALCOLO

$$G_{fd} = \frac{G_{fd}}{\gamma_g} \quad \varepsilon_{fdm} = \frac{2k_{cr}}{\gamma_{Rd}} \sqrt{\frac{k(n)G_{fd}}{E_f t_{1f}}} \quad P_{fde} = \frac{2nb_f}{\gamma_{Rd}} \sqrt{k(n) \cdot E_f t_{1f} G_{fd}}$$

RESISTENZA A TAGLIO



- Shear strength

$$V_{Rd} = \min \{ V_{wd} + V_{ctd} + V_{fd}, V_{cd} \}$$

External strengthening

V_{cd} : diagonal concrete crushing

V_{wd} : steel contribution

V_{fd} : external strengthening contribution

V_{ctd} : concrete contribution

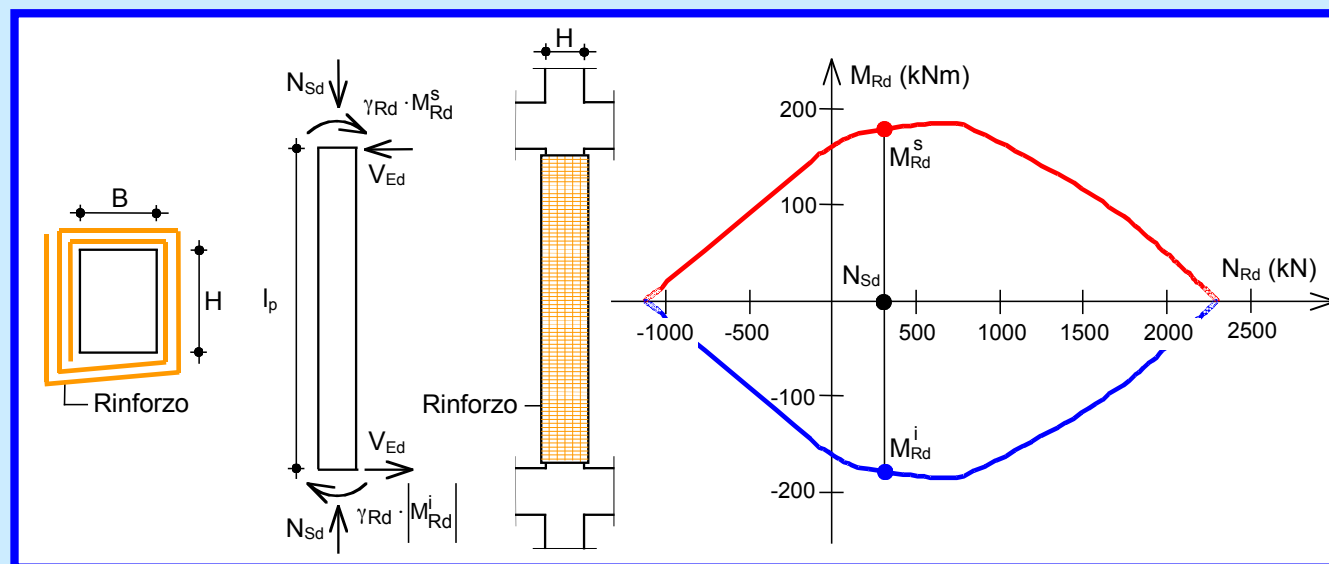
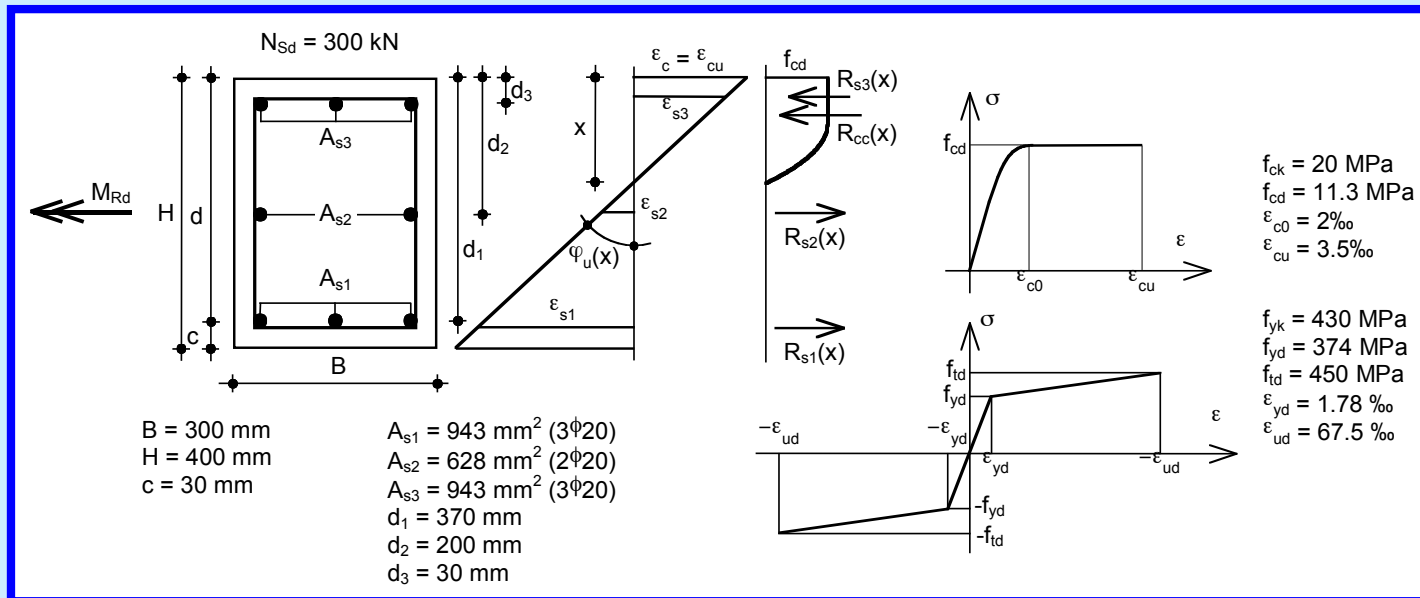
$$V_f = \frac{1}{\gamma_{Rd}} 0.9 \cdot d \frac{2t_f w_f}{i_f} \cdot E_f \varepsilon_{fdv} \cdot \sin \beta \cdot (\cot \theta + \cot \beta)$$

$$\varepsilon_{fdv} = \phi \cdot \varepsilon_{fde}$$

$$\phi = 1 - \frac{1}{3} \frac{L_e}{h_w} \cdot \sin \beta$$

Dilatazione di delaminazione di estremità
Lunghezza di trasferimento efficace

ESEMPIO (GR)



$$V_{Ed} = \gamma_{Rd} \frac{|M_{Rd}^s| + |M_{Rd}^i|}{l_p} = 1.1 \frac{178.7 + 178.7}{3} = 131.05 \text{ kN}$$

ESEMPIO (GR)

- Senza rinforzo (trascurando il contributo del calcestruzzo)

$$V_{Rd0} = V_{Rds} = 0.9 \cdot d \cdot \frac{A_{sw}}{i_s} \cdot f_{yd} = 0.9 \cdot 0.37 \cdot \frac{1.005 \cdot 10^{-4}}{0.2} \cdot 3.74 \cdot 10^5 = 62.58 \text{ kN}$$

- Energia di frattura di calcolo dell'interfaccia

$$G_{fd} = \frac{G_f}{\gamma_g} = \frac{211}{1.35} = 156 \frac{\text{J}}{\text{m}^2}$$

- Dilatazione di delaminazione di calcolo del rinforzo

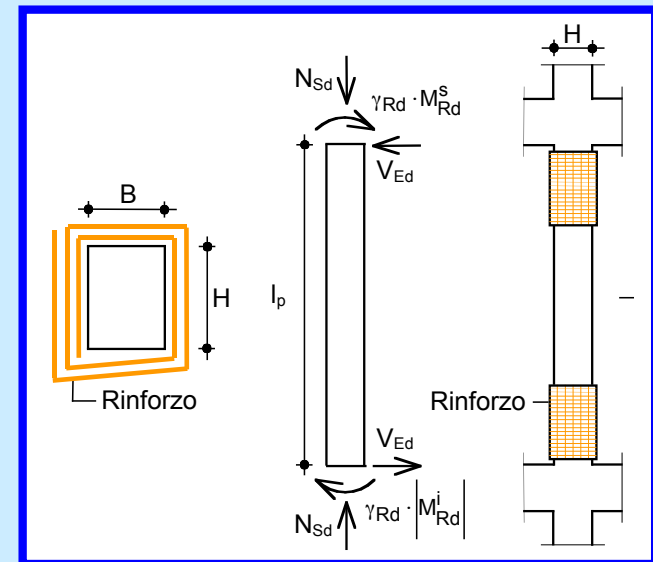
$$\varepsilon_{fde} = \frac{2}{\gamma_{Rd}} \sqrt{\frac{k(n)G_{fd}}{E_f t_{1f}}} = \frac{2}{1.2} \sqrt{\frac{0.8 \cdot 0.156}{270000 \cdot 0.045}} = 5.3 \text{‰}$$

- Contributo del rinforzo

$$V_{Rdf} = \frac{0.9 \cdot d}{\gamma_{Rdv}} \cdot 2t_f \cdot E_f \cdot \varepsilon_{fdv} = \frac{0.9 \cdot 370}{1.2} \cdot 2 \cdot 0.09 \cdot \frac{270000}{1000} \cdot \frac{5.3}{1000} = 71.48 \text{ kN}$$

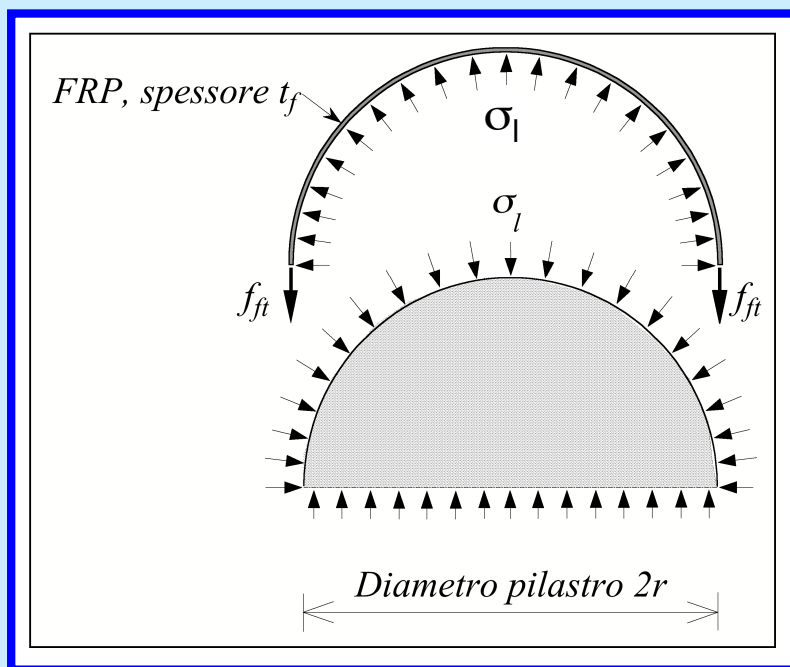
- Resistenza a taglio

$$V_{Rd} = V_{Rds} + V_{Rdf} = 62.58 + 71.48 = 134.07 \text{ kN} > V_{Ed} = 131.05 \text{ kN}$$



CONFINAMENTO

- Sezione circolare con fasciatura uniforme lungo il fusto secondo CNR DT200



➡ Deformazione ridotta di calcolo:

$$\varepsilon_{fd,rid} = \min \left\{ \frac{\eta_a \varepsilon_{fu}}{\gamma_f}, 0.004 \right\}$$

Dilatazione di rottura delle fibre

➡ Tensione efficace del rinforzo:

$$f_{ft} = E_f \cdot \varepsilon_{fd,rid}$$

➡ Pressione di confinamento (a “rottura” del composito)

$$\sigma_l = \frac{f_{ft} t_f}{R} \quad (\text{per l'equilibrio nelle condizioni di rottura})$$

➡ Resistenza con confinamento:

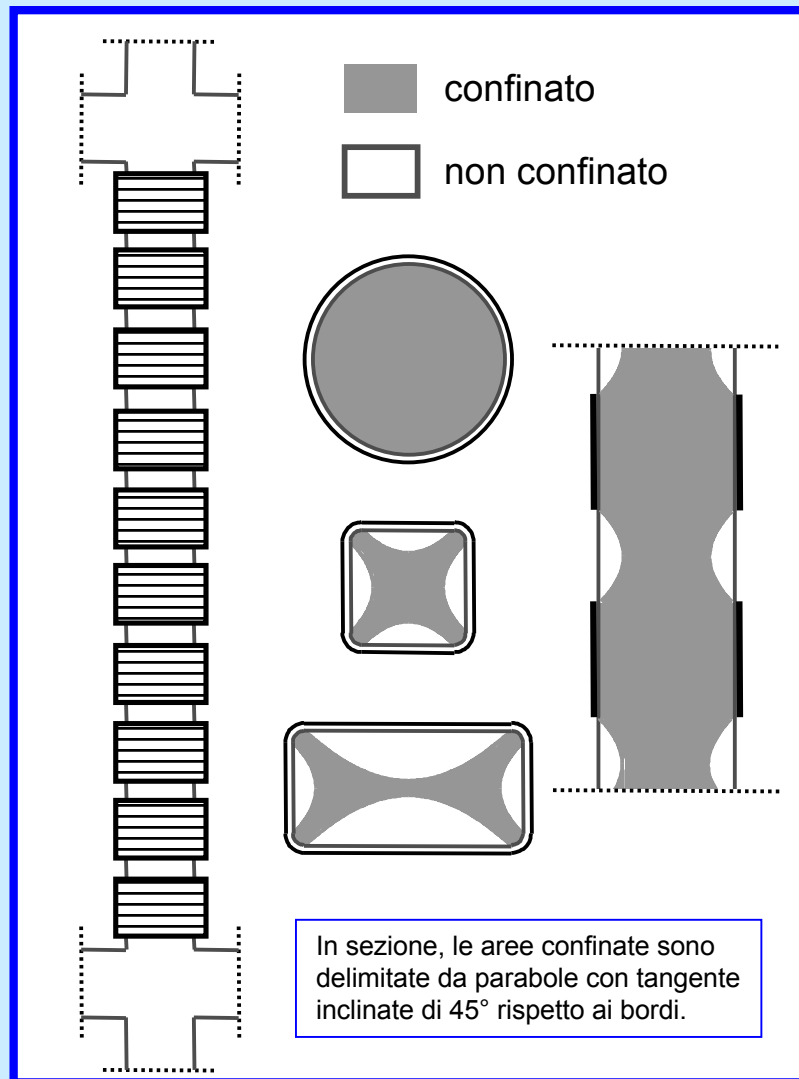
$$f_{ccd} = f_{cd} \cdot \left[1 + 2.6 \cdot \left(\frac{\sigma_l}{f_{cd}} \right)^{\frac{2}{3}} \right]$$

$\gamma_f = 1.2 \div 1.35$ coefficiente parziale di sicurezza (dipendente dalle condizioni di controllo di esecuzione);

$\eta_a = 0.5 \div 0.95$ dipendente dal sistema (tipo di fibre e matrice) e dalle condizioni di esposizione.

CONFINAMENTO

- CASO GENERALE (sezione rettangolare con fasciatura non uniforme)



→ Vale quanto visto prima, salvo considerare la “pressione efficace di confinamento”, definita da:

$$\sigma_1' = k_{\text{eff}} \cdot \sigma_1$$

→ k_{eff} (coefficiente di efficienza) si valuta come il rapporto tra il volume efficacemente confinato ed il volume totale:

$$k_{\text{eff}} = \frac{V_{\text{conf}}}{V_{\text{tot}}}$$

→ in via semplificata:

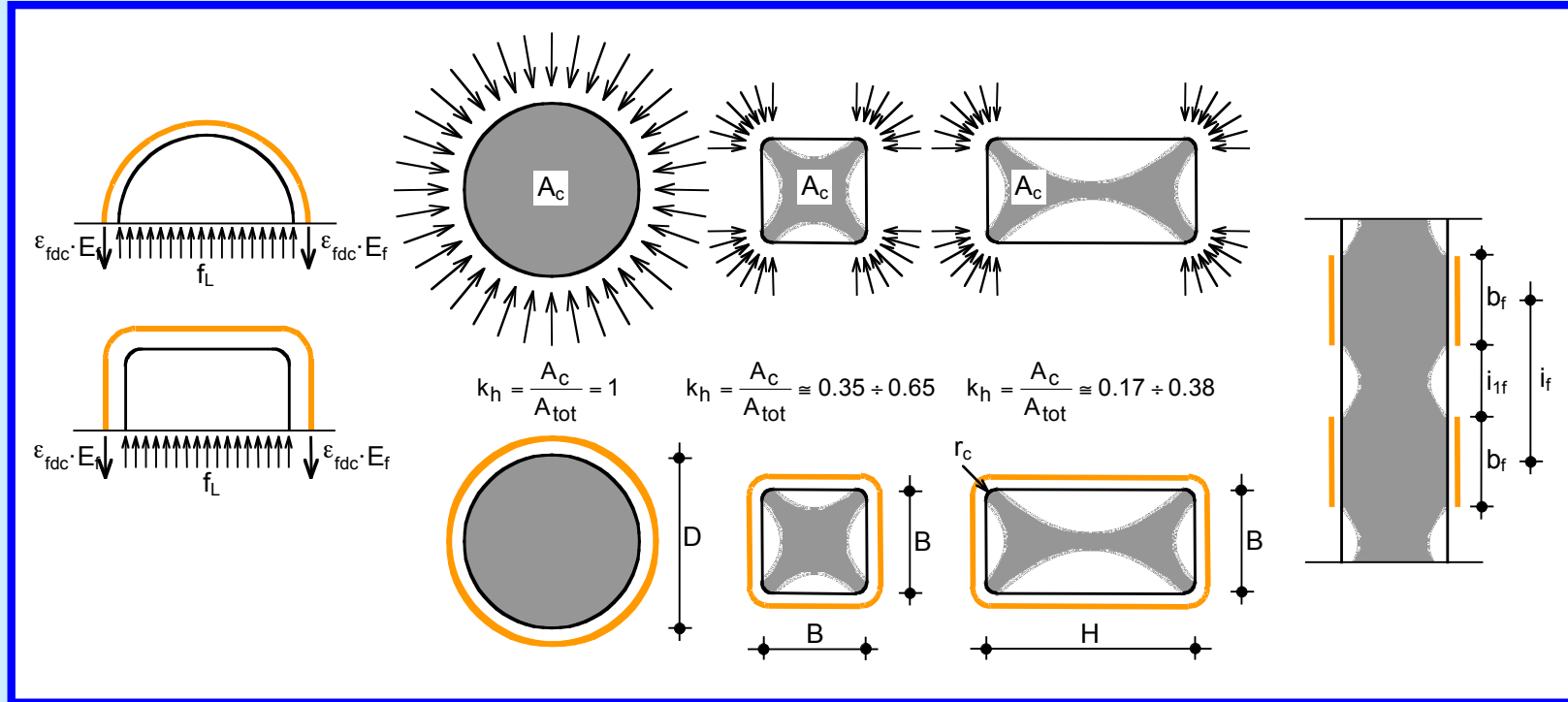
$$k_{\text{eff}} = k_v k_h$$

k_v : coeff. di efficienza verticale

k_h : coeff. di efficienza orizzontale

CONFINAMENTO

- CASO GENERALE (sezione rettangolare con fasciatura non uniforme)



$$\sigma_l' = k_{eff} \cdot \sigma_l$$

$$k_v = \left(1 - \frac{i_{1f}}{2 \cdot D}\right)^2$$

sezione
circolare

$$k_v = \left(1 - \frac{i_{1f}}{2 \cdot b}\right)^2$$

sezione
rettangolare ($b < a$)

➔ Per la sezione rettangolare, la pressione laterale di confinamento (media) si valuta come:

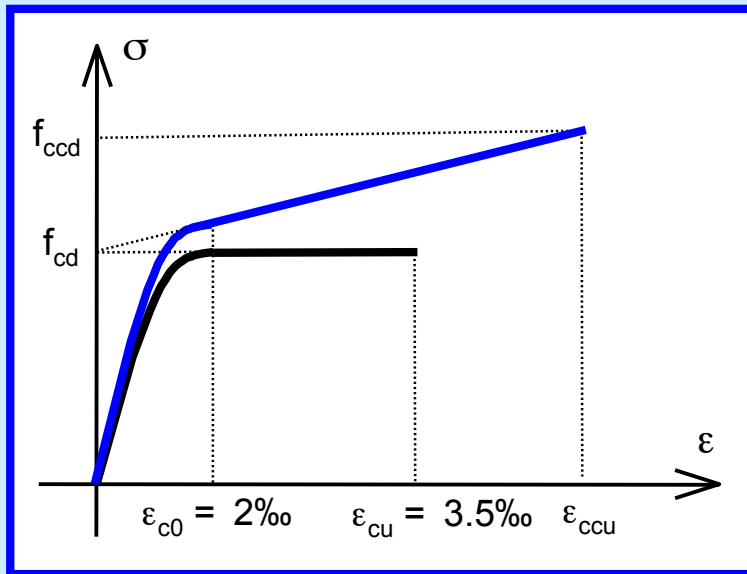
$$\sigma_l = \frac{t_f (a + b)}{ab} \cdot \frac{b_f}{p_f} \cdot E_f \cdot \epsilon_{fd,rid}$$

CONFINAMENTO

- INCREMENTO DI DUTTILITÀ

➡ La fasciatura con compositi aumenta anche la duttilità di **elementi pressoinflessi**.

➡ CNR DT200, in alternativa il legame costitutivo non lineare definito da:



$$\sigma_{cc}(\varepsilon) = \begin{cases} f_{cd} \cdot \left[A \cdot \frac{\varepsilon}{\varepsilon_{c0}} - \left(\frac{\varepsilon}{\varepsilon_{c0}} \right)^2 \right] & \text{se } 0 \leq \varepsilon \leq \varepsilon_{c0} \\ f_{cd} \cdot \left(1 + B \frac{\varepsilon}{\varepsilon_{c0}} \right) & \text{se } \varepsilon_{c0} < \varepsilon \leq \varepsilon_{ccu} \end{cases}$$

$$A = 1 + \frac{f_{cd} + E_t \cdot \varepsilon_{c0}}{f_{cd}}$$

$$B = \frac{f_{cd} + E_t \cdot \varepsilon_{c0}}{f_{cd}} - 1$$

$$E_t = \frac{f_{ccd} - f_{cd}}{\varepsilon_{ccu}}$$

$$f_{ccd} = f_{cd} \cdot \left[1 + 2.6 \cdot \left(\frac{\sigma_l}{f_{cd}} \right)^{\frac{2}{3}} \right]$$

$$\varepsilon_{ccu} = 0.0035 + 0.015 \sqrt{\frac{\sigma_l}{f_{cd}}}$$

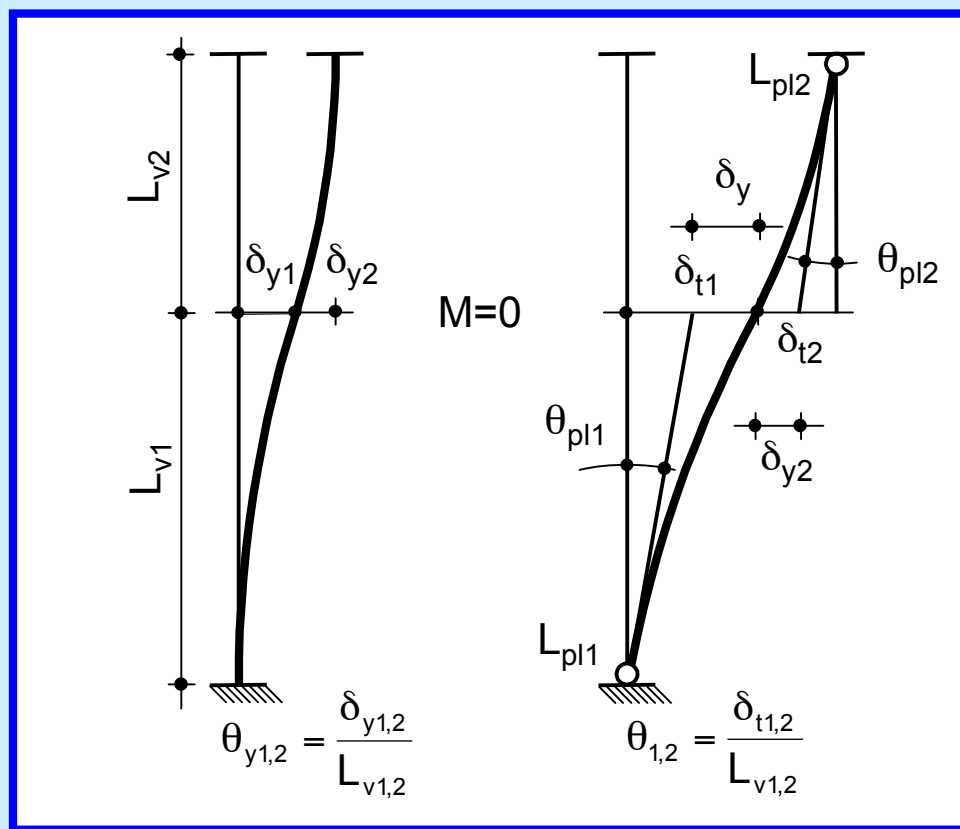
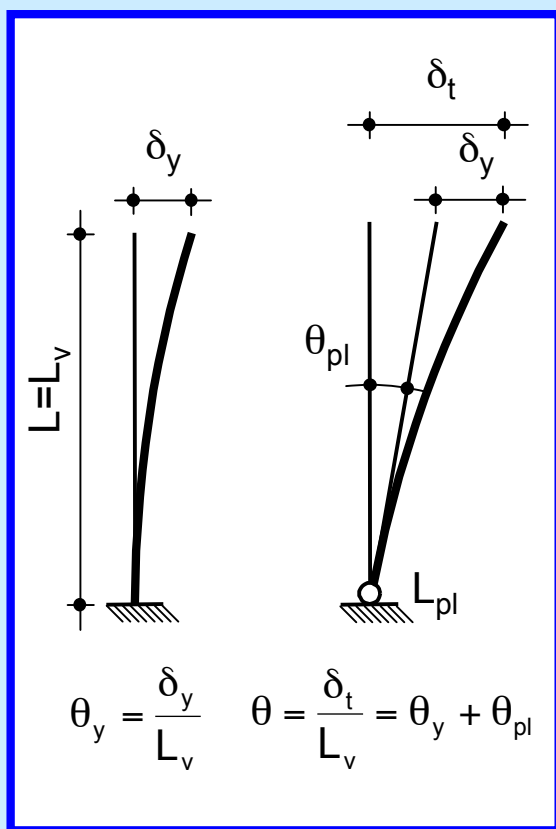
• CAPACITA' DI ROTAZIONE RISPETTO ALLA CORDA

$$\theta_u = \frac{1}{\gamma_{el}} \cdot \left[\theta_y + (\phi_u - \phi_y) \cdot L_{pl} \cdot \left(1 - \frac{L_{pl}}{2 \cdot L_v} \right) \right]$$

SL di CO: $\theta \leq \theta_u$

SL di DS: $\theta \leq \theta_{DS} = \frac{3}{4} \theta_u$

Stessa formula usata in assenza di rinforzo, **CAMBIA** ϕ_u



• CAPACITA' DI ROTAZIONE RISPETTO ALLA CORDA

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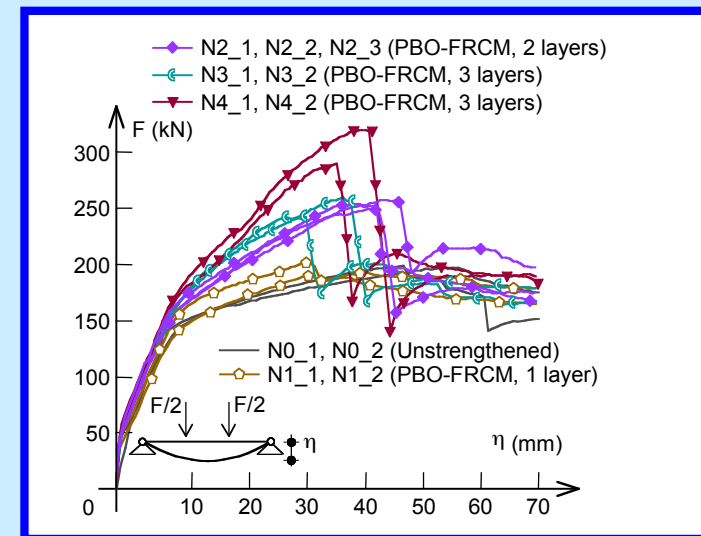
Stessa formula usata in assenza di rinforzo,
CAMBIA ϕ_u

$$\theta_y = \phi_y \cdot \frac{L_v}{3} + 0.0013 \cdot \left(1 + 1.5 \cdot \frac{h}{L_v} \right) + 0.13 \cdot \phi_y \cdot \frac{d_b \cdot f_y}{\sqrt{f_c}}$$

Stessa formula usata
in assenza di rinforzo

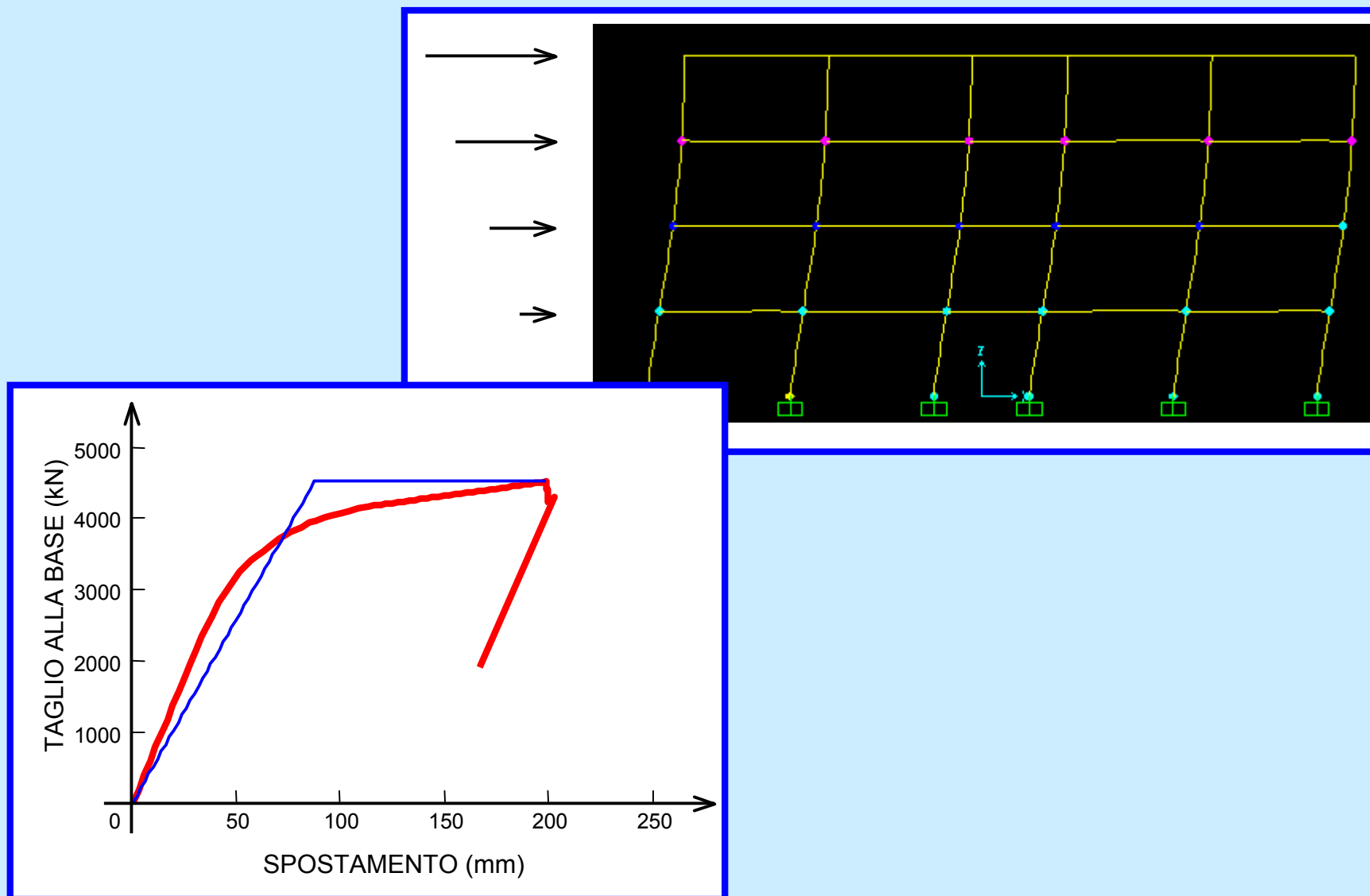
$$L_{pl} = 0.1 \cdot L_v + 0.17 \cdot h + 0.24 \cdot \frac{d_b \cdot f_y}{\sqrt{f_c}}$$

Lunghezza della cerniera plastica



- **CAPACITA'** DI ROTAZIONE RISPETTO ALLA CORDA

Utilizzata nell'analisi statica non lineare (pushover)



ESEMPIO

$$\varepsilon_{fd,rid} = \varepsilon_{fde} = 5.3 \text{ ‰}$$

$$f_{ck} = 20 \text{ MPa}$$

$$f_{cd} = \frac{\alpha_c}{\gamma_c} \cdot f_{ck} = \frac{0.85}{1.5} \cdot 20 = 11.3 \text{ MPa}$$

- Coefficienti di efficienza

$$k_h = 1 - \frac{(B - 2r_c)^2 + (H - 2r_c)^2}{3 \cdot A_g} =$$

$$= 1 - \frac{(300 - 60)^2 + (400 - 60)^2}{3 \cdot \left[300 \cdot 400 - \left(1 - \frac{\pi}{4}\right) \cdot 30^2 \right]} = 0.516$$

$$k_v = \left(1 - \frac{i_{1f}}{2 \cdot d_{min}}\right)^2 = 1$$

- Pressione di confinamento

$$f_l = \frac{t_f (B + H)}{BH} \cdot \frac{b_f}{i_f} \cdot E_f \cdot \varepsilon_{fd,rid} = 0.751 \text{ MPa}$$

$$t_f = 2 \cdot 0.045 = 0.09 \text{ mm}$$

- Pressione di confinamento efficace

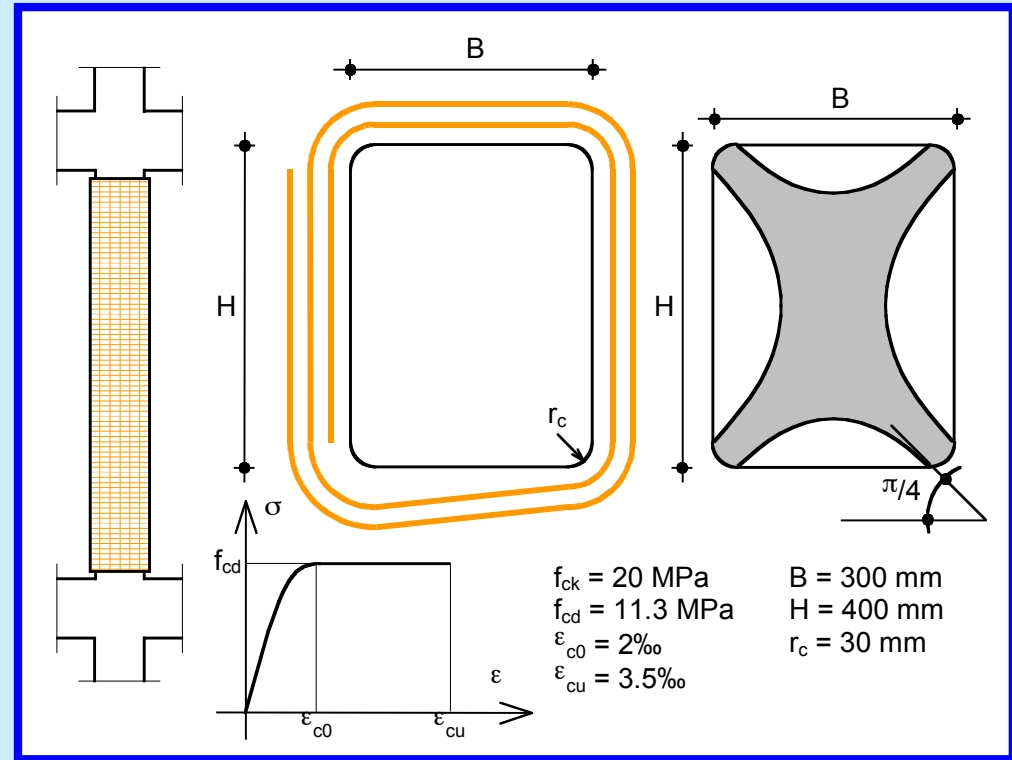
$$f_{Leff} = k_{eff} \cdot f_l = k_h \cdot k_v \cdot f_l = 0.751 \cdot 1 \cdot 0.516 = 0.387 \text{ MPa}$$

- Resistenza con confinamento

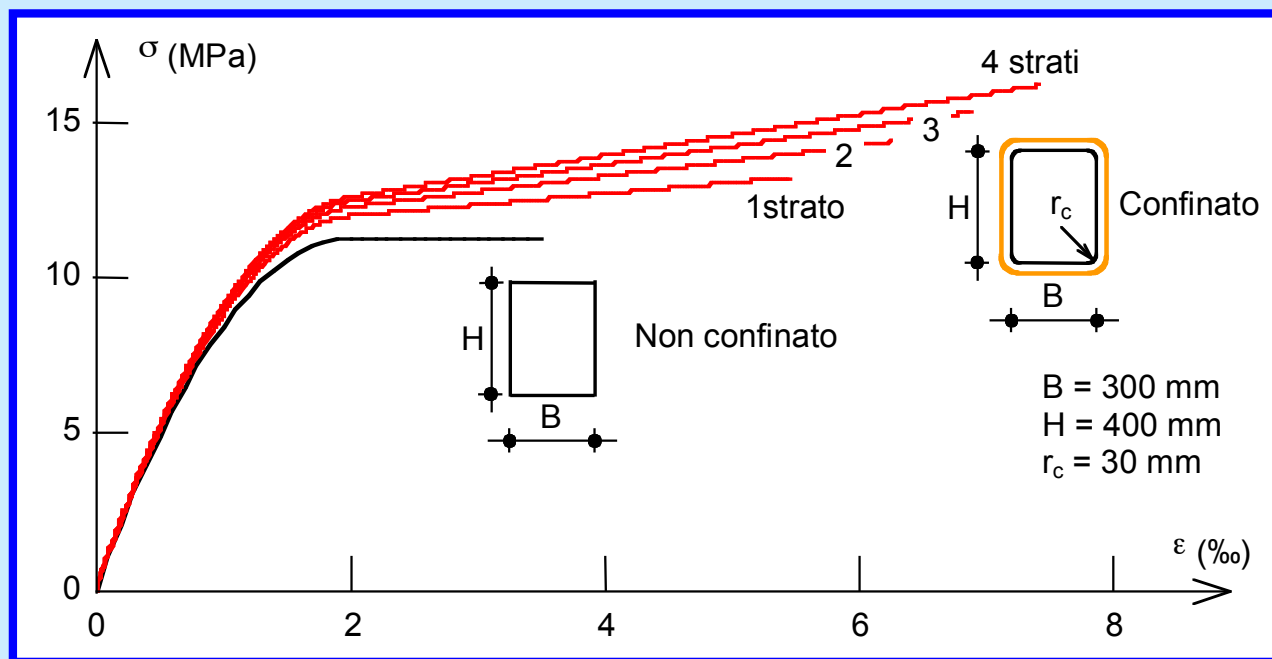
$$f_{ccd} = f_{cd} \cdot \left[1 + 2.6 \cdot \left(\frac{f_{Leff}}{f_{cd}} \right)^{\frac{2}{3}} \right] = 11.3 \cdot \left[1 + 2.6 \cdot \left(\frac{0.387}{11.3} \right)^{\frac{2}{3}} \right] = 11.3 \cdot 1.275 = 14.4 \text{ MPa}$$

- Deformazione ultima

$$\varepsilon_{ccu} = 0.0035 + 0.015 \cdot \sqrt{\frac{f_{Leff}}{f_{cd}}} = 0.0035 + 0.015 \cdot \sqrt{\frac{0.387}{11.3}} = 6.27 \text{ ‰}$$



ESEMPIO



	Resistenza a compressione, f_{ccd} (MPa)	f_{ccd}/f_{cd}	Deformazione ultima ε_{ccu} (‰)	$\varepsilon_{ccu}/\varepsilon_{cu}$
Non rinforzato	11.3	--	3.50	--
1 strato	13.3	1.17	5.46	1.56
2 strati	14.4	1.27	6.27	1.79
3 strati	15.4	1.36	6.90	1.97
4 strati	16.3	1.43	7.42	2.12

ESEMPIO

$$N_{Sd} = 350 \text{ kN}$$

- Curvatura ultima

$$\varphi_{uR}(x) = \frac{\varepsilon_{ud}}{d-x} \quad \text{Rottura lato teso}$$

$$\varphi_{uR}(x) = \frac{\varepsilon_{ccu}}{x} \quad \text{Rottura lato compresso}$$

- Equilibrio alla traslazione

$$R_{cc}(x) + R_{sc}(x) - R_{st}(x) = N_{Sd}$$

$$R_{sc}(x) = A_{s2} \cdot \sigma_s(\varphi_{uR}(x) \cdot (x - c))$$

$$R_{st}(x) = A_{s1} \cdot \sigma_s(\varphi_{uR}(x) \cdot (d - x))$$

$$R_{cc}(x) = B \cdot \int_0^x \sigma_{cc}(\varphi_{uR}(x) \cdot z) dz$$

- Posizione dell'asse neutro

$$x = 101.5 \text{ mm} \quad \text{Confinato}$$

$$x = 127.8 \text{ mm} \quad \text{Non confinato}$$

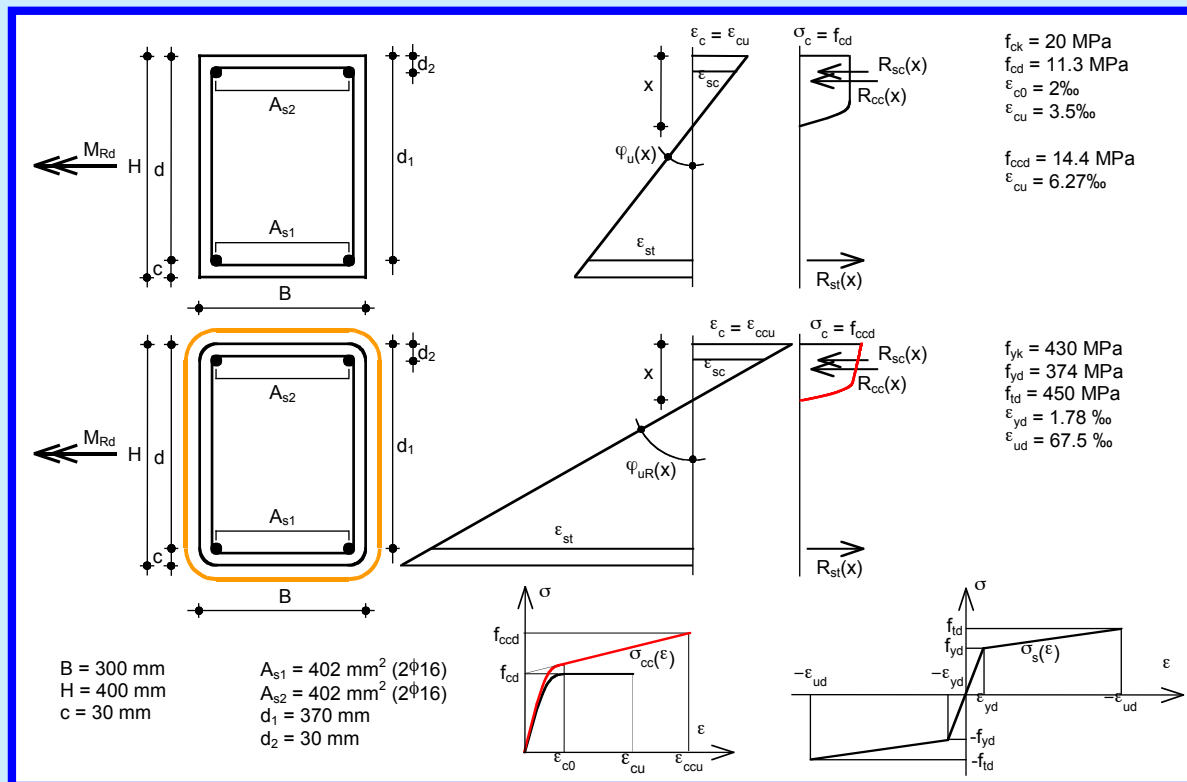
- Momento ultimo

$$M_{Rd} = B \cdot \int_0^x \sigma_{cc}(\varphi_{uR}(x) \cdot z) \cdot \left(\frac{H}{2} - x + z \right) dz +$$

$$+ A_{s2} \cdot \sigma_s(\varphi_{uR}(x) \cdot (x - c)) \cdot \left(\frac{H}{2} - c \right) + A_{s1} \cdot \sigma_s(\varphi_{uR}(x) \cdot (d - x)) \cdot \left(d - \frac{H}{2} \right)$$

$$M_{Rd} = 107.9 \text{ kNm} \quad \text{Confinato}$$

$$M_{Rd} = 103.2 \text{ kNm} \quad \text{Non confinato}$$



ESEMPIO

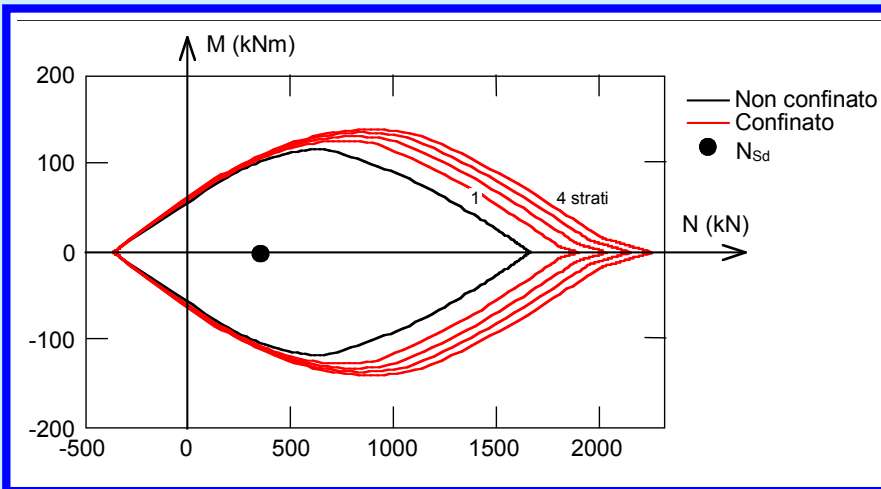
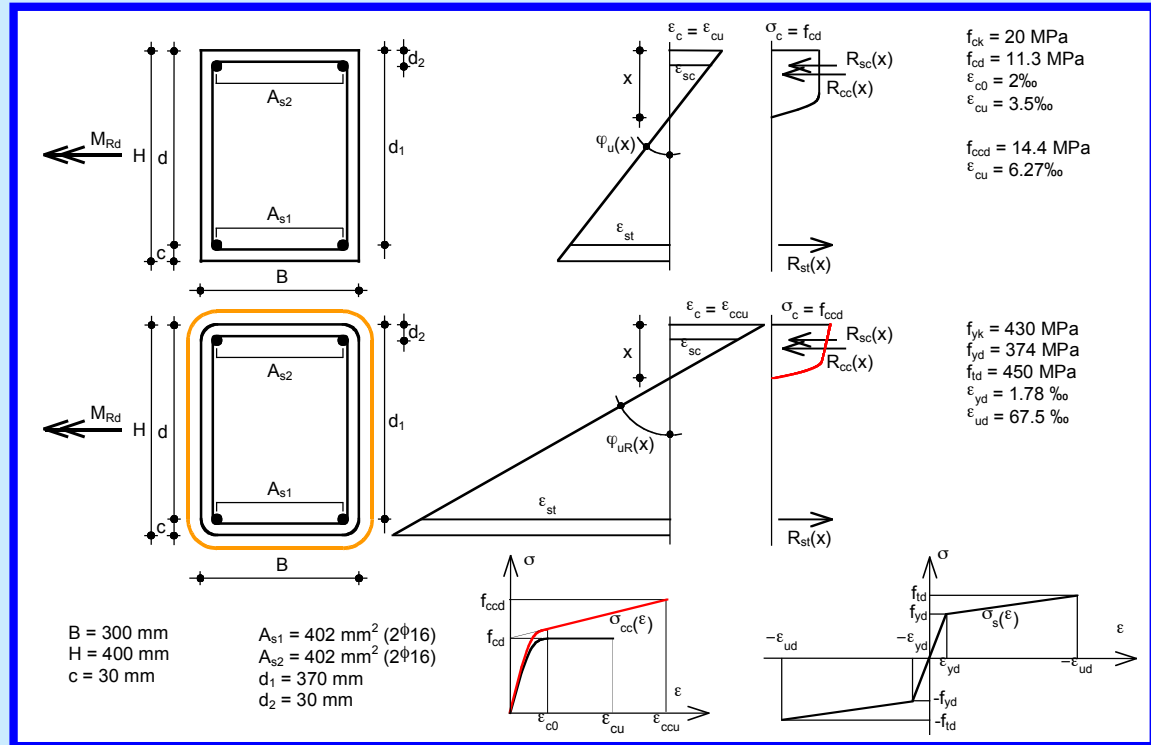
Curvatura ultima

Non confinato

$$\varphi_u(x) = \frac{\varepsilon_{cu}}{x} = \frac{3.5}{1000 \cdot 0.1278} = 0.027 \frac{1}{m}$$

Confinato

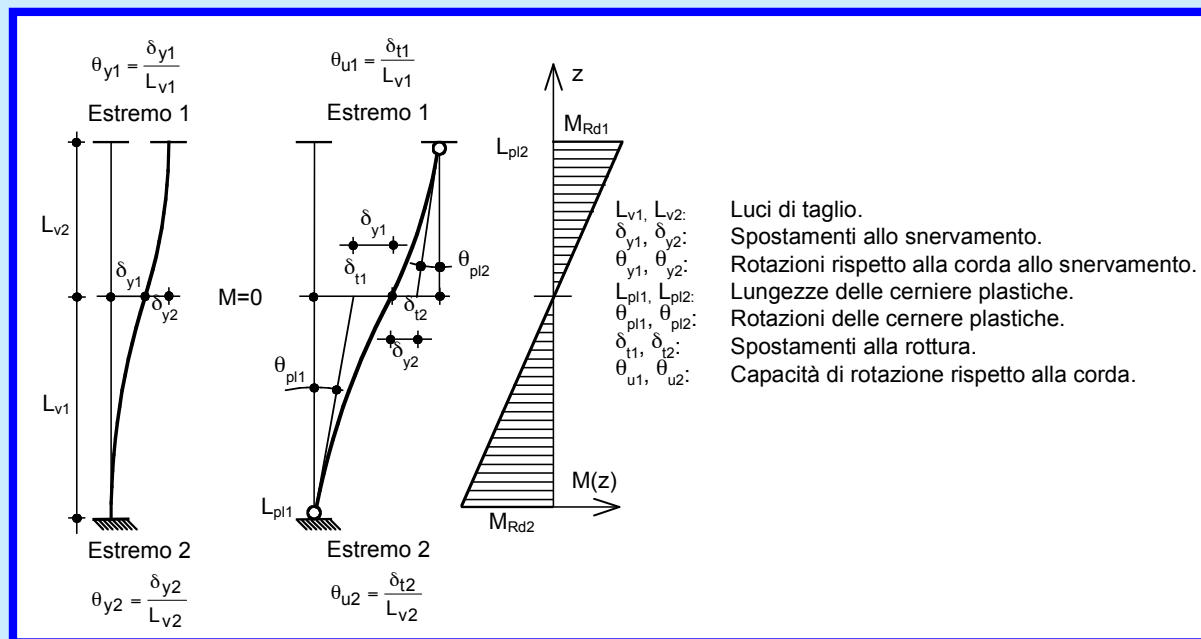
$$\varphi_{uR}(x) = \frac{\varepsilon_{ccu}}{x} = \frac{6.28}{1000 \cdot 0.1014} = 0.062 \frac{1}{m}$$



	M_{Rd} (kNm)	M_{Rd}/M_{Rd0}	φ_{uR} (1/m)	$\varphi_{uR}/\varphi_{uR0}$
Non rinforzato	103.3	--	0.027	--
1 strato	106.4	1.03	0.051	1.85
2 strati	107.9	1.05	0.062	2.26
3 strati	109.1	1.06	0.071	2.60
4 strati	110.0	1.07	0.080	2.91

• CAPACITA' DI ROTAZIONE RISPETTO ALLA CORDA

$$\theta_u = \frac{1}{\gamma_{el}} \cdot \left[\theta_y + (\phi_u - \phi_y) \cdot L_{pl} \cdot \left(1 - \frac{L_{pl}}{2 \cdot L_v} \right) \right]$$



• Cerniera plastica

$$L_{pl} = 0.1 \cdot L_v + 0.17 \cdot H + 0.24 \cdot \frac{d_b f_y}{\sqrt{f_c}} = 0.1 \cdot 1.5 + 0.17 \cdot 0.4 + 0.24 \cdot \frac{0.016 \cdot 374}{\sqrt{11.3}} = 0.64 \text{ m} \quad \text{Non confinato}$$

$$L_{pl} = 0.1 \cdot L_v + 0.17 \cdot H + 0.24 \cdot \frac{d_b f_y}{\sqrt{f_c}} = 0.1 \cdot 1.5 + 0.17 \cdot 0.4 + 0.24 \cdot \frac{0.016 \cdot 374}{\sqrt{14.4}} = 0.60 \text{ m} \quad \text{Confinato}$$

- CAPACITA' DI ROTAZIONE RISPETTO ALLA CORDA

$$\theta_u = \frac{1}{\gamma_{el}} \cdot \left[\theta_y + (\phi_u - \phi_y) \cdot L_{pl} \cdot \left(1 - \frac{L_{pl}}{2 \cdot L_v} \right) \right]$$

- Curvatura e momento allo snervamento

Non confinato	$\varphi_y = 0.0093 \frac{1}{m}$	Confinato	$\varphi_y = 0.0095 \frac{1}{m}$
	$M_y = 97.29 \text{ kNm}$		$M_y = 96.72 \text{ kNm}$

- Rotazione rispetto alla corda allo snervamento

$$\begin{aligned} \theta_y &= \varphi_y \cdot \frac{L_v}{3} + 0.0013 \cdot \left(1 + 1.5 \cdot \frac{H}{L_v} \right) + 0.13 \cdot \varphi_y \frac{d_b f_y}{\sqrt{f_c}} = \\ &= 0.0093 \cdot \frac{1.5}{3} + 0.0013 \cdot \left(1 + 1.5 \cdot \frac{0.4}{1.5} \right) + 0.13 \cdot 0.0093 \cdot \frac{0.016 \cdot 374}{\sqrt{14.4}} = 8.36 \cdot 10^{-3} = 0.48^\circ \quad \text{Confinato} \end{aligned}$$

$$\begin{aligned} \theta_y &= \varphi_y \cdot \frac{L_v}{3} + 0.0013 \cdot \left(1 + 1.5 \cdot \frac{H}{L_v} \right) + 0.13 \cdot \varphi_y \frac{d_b f_y}{\sqrt{f_c}} = \\ &= 0.0095 \cdot \frac{1.5}{3} + 0.0013 \cdot \left(1 + 1.5 \cdot \frac{0.4}{1.5} \right) + 0.13 \cdot 0.0095 \cdot \frac{0.016 \cdot 374}{\sqrt{11.3}} = 8.77 \cdot 10^{-3} = 0.50^\circ \quad \text{Non confinato} \end{aligned}$$

